

AIM: How do we determine max/min values?

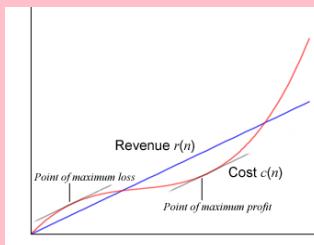
Warm Up:

- 1) Find 2 positive numbers whose sum is 40 and whose product is 175.

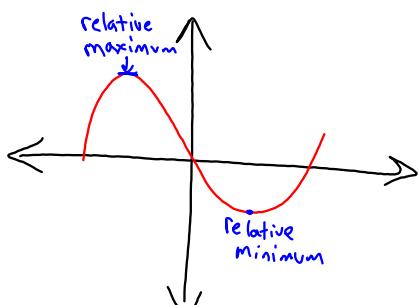
$$\begin{aligned} X+Y &= 40 & XY &= 175 \\ X &= 40-Y & Y(40-Y) &= 175 \\ & & 40Y - Y^2 &= 175 \\ 0 &= Y^2 - 40Y + 175 & & \\ 0 &= (Y-35)(Y-5) & & \\ Y &= 35 & Y &= 5 \end{aligned}$$

5 & 35

Nov 14-8:33 AM



Jan 12-9:44 AM



Feb 2-8:00 AM

Optimization Problems - Classwork

Many times in life we are asked to do an optimization problem - that is, find the largest or smallest value of some quantity that will fulfill a need. Typical situations are:

- build the house which will minimize the time it takes me to get to school.
- build a structure costing the least amount of material.
- build a yard covering the most amount of area.
- find the most money you can earn by helping a medical problem.
- find how the most ones should change for a CD in order to make as much money as possible.

All of these situations have something in common - they are all trying to maximize or minimize some quantity. This lends itself to a calculus solution. We have spent the better part of last month trying to find maximum and minimum values of functions. Now we can apply that knowledge to real world problems where we want to maximize or minimize. So in solving word problems, you must look carefully for certain words among all the verbiage. Look for words like "maximize and", "smallest volume", "least amount of time", "shortest distance", "Closest", etc. These words will tell you what kind of problem it is.

1. **Assign variables** to all given quantities and quantities to be determined. Don't be afraid to use letters you usually do not use (h, o, g, etc.). When feasible, make a sketch of the problem.
2. Making a chart of possible answers allows you to see a relationship between variables. While not necessary, it is helpful.
3. **Write a "primary" equation** for the quantity you found needs to be maximized or minimized.

Area of Rectangle = length • width	Hypotenuse = $\sqrt{x^2 + y^2}$
Distance = rate • time	Perimeter of a rectangle = $2x + 2y$
Volume of cylinder = $\pi(r^2)h$	Volume of cylinder = $\pi(r^2)h$
4. Reduce the right side of this "primary equation" to one having a single variable. If there is more than one variable on the right side, you must write a "secondary" equation (a restriction or constraint) relating the variables of the primary equation.
5. Take the derivative of the equation and set equal to zero. If you get more than one answer, make a sign chart to determine whether it represents a maximum or minimum. Pay attention to whether that value makes sense. Time is either negative (it can't take negative 7 hours to run a race). You cannot use more than you have (you can't have a length of 8 feet when you only have 6 feet of fencing).
6. Be sure that you answer the question that is asked. If you are asked to find a minimum or maximum value of both quantity, you must plug your answer from (4) into your primary equation.
7. If you are to find a maximum or minimum on a closed interval, you must test the endpoints as well. Make sure your work is clear.
8. You can verify your answers by graphing your primary equation with one variable on the calculator. Use your 2nd + CALC: maximum or minimum function.

MasterMathMentor.com

- 108 -

Suz Schwartz

Feb 3-10:22 AM

sample 1) Two numbers add up to 40. Find the numbers that maximize their product.

Smaller Number	1	13	19	20	
Larger Number	39	27	21	20	
Product	39	133	371	400	

$$\begin{aligned} XY &= MAX \\ X(40-X) &= Product \\ 40X - X^2 &= Product \\ 40 - 2X &= P \\ 40 - 2X &= 0 \\ 2X &= 40 \\ X &= 20 \\ Y &= 20 \end{aligned}$$

Verify that this is a max

Jan 12-9:33 AM

Example 2) A rectangle has a perimeter of ~~48~~ feet. What length and width should it have so that its area is a maximum? What is this maximum area?

$$\begin{aligned} XY &= Area \\ X(36-X) &= A \\ 36X - X^2 &= A \\ 36 - 2X &= A \\ 36 - 2X &= 0 \\ 2X &= 36 \\ X &= 18 \\ Y &= 18 \end{aligned}$$

*Length 18 ft
Width 18 ft*

$\frac{A}{18} = \frac{1}{2}(36 - 2X)$

$\frac{A}{18} = \frac{1}{2}(36 - 2 \cdot 18)$

$\frac{A}{18} = \frac{1}{2}(36 - 36)$

$\frac{A}{18} = \frac{1}{2}(0)$

$A = 0$

Max at $X=18$

$0 \leq L \leq 36$

$0 \leq W \leq 36$

$Max\ Area: 324\ ft^2$

Jan 12-9:36 AM

(*) If we have a closed interval we can use the closed interval theorem like when we did Absolute min/Max.

- Test critical values and endpoints of the interval into the original function.

Jan 12-9:36 AM

Example 3) Find two positive numbers that minimize the sum of twice the first number plus the second if the product of the two numbers is 288.

<u>Primary</u> $S = 2x + y$ $S = 2x + \frac{288}{x}$ $S = 2x + 288x^{-1}$ $S' = 2 - 288x^{-2}$ $O = 2 - \frac{288}{x^2}$ $\frac{288}{x^2} = 2$ $x^2 = \frac{288}{2}$ $x^2 = 144$ $x = \pm 12$ $x = 12$	<u>Secondary</u> $xy = 288$ $y = \frac{288}{x}$ $x > 0$ $y > 0$ $S \text{ number line}$ $\begin{array}{c} S \\ \hline - + \end{array}$ $\text{Min } \partial x = 12$ $y = 24$ $12 \cdot 24$
--	--

Feb 4-10:33 AM

sample 4) An open box is to be made from a piece of metal 16 by 30 inches by cutting out squares of equal size from the corners and bending up the sides. What size square should be cut out to create a box with greatest volume. What is the maximum volume as well?

Primary

$$V = lwh$$

$$V(x) = (30-2x)(16-2x)x$$

$$V(x) = 480x - 92x^2 + 4x^3$$

$$V'(x) = 12x^2 - 184x + 480 \quad 0 < x < 15$$

Use quad formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{184 \pm \sqrt{184^2 - 4 \cdot 12 \cdot 480}}{2 \cdot 12}$$

$$x = \frac{184 \pm \sqrt{144}}{24}$$

$$x = \frac{184 \pm 12}{24}$$

$$x = 10/3 \quad x = 12$$

MAX volume $\partial x = 10/3$

$$V(10/3) = 480\left(\frac{10}{3}\right) - 92\left(\frac{10}{3}\right)^2 + 4\left(\frac{10}{3}\right)^3$$

$$= 725.926 \text{ in}^3$$

Feb 4-10:34 AM

- Steps for Maximizing/Minimizing
- 1) Identify the function that we are getting the max/min of (Primary)
 - 2) Find the derivative of the Primary
 - 3) Set derivative = 0 and solve. (If derivative is a fraction solve for undefined)
 - 4) Use First derivative test (number line) to verify the max or min.
 - 5) Answer the question.

Feb 8-10:37 AM

1. A farmer has 600 m of fencing with which he plans to enclose a rectangular pen adjacent to a long existing wall. He will use the wall for one side of the pen and the available fencing for the three remaining sides. What is the maximum area that he can enclose this way?

Primary

$$A = xy$$

$$A = x(600 - 2x)$$

$$A = 600x - 2x^2$$

$$A' = 600 - 4x$$

$$4x = 600$$

$$x = 150$$

$$A = 150(300)$$

$$= 45,000 \text{ m}^2$$

Secondary

$$0 < x < 300$$

$$2x + y = 600$$

$$y = 600 - 2x$$

$$600 - 2x > 0$$

$$-2x > -600$$

$$x < 300$$

Number Line

$$\begin{array}{c} x \\ \hline - + \end{array}$$

$$\text{MAX } \partial x = 150$$

Feb 8-10:45 AM