Name _____

Optimization Worksheet #1

1. The sum of a number and 3 times another number is 12. Their product is a maximum. Find the numbers and the max product.

Primary

$xy = P$

$(12-3y)y = P$

$12y - 3y^2 = P$

$12 - 6y = P'$

Secondary

$x + 3y = 12$

$x = 12 - 3y$

P': + 2 −

Max @ $x = 2$

$12 - 6y = 0$

$12 = 6y$

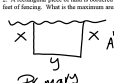
$y = 2$

$x = 12 - 3(2)$

$x = 6$

Max product 12

2. A rectangular piece of land is bordered on one side by a river. The other 3 sides are to be enclosed by 300 feet of fencing. What is the maximum area that can be enclosed?



Secondary

$2x + y = 300$

$y = 300 - 2x$

$A' = 300 - 4x$

$4x = 300$

$x = 75$

$y = 150$

$300 - 2x > 0$

$-2x > -300$

$x < 150$

$0 < x < 150$

Primary

$A = xy$

$A = x(300 - 2x)$

$A = 300x - 2x^2$

A': + 75 −

Max Area is $11250\,ft^2$

3. An open rectangular box is made by cutting 4 congruent squares from the corners of a rectangular piece of cardboard and folding the sides up. The cardboard is 12 feet on each side. What are the dimensions of the box with the maximum volume?

$V_{max} = lwh$

$V = (12-2x)(12-2x)x$

$V = 144x - 48x^2 + 4x^3$

$V' = 144 - 96x + 12x^2$

$0 = 144 - 96x + 12x^2$

$0 = 12(x^2 - 8x + 12)$

$0 = 12(x-6)(x-2)$

$x \neq 6 \mid x = 2$

$x > 0$

$12 - 2x > 0$

$-2x > -12$

$x < 12$

$0 < x < 6$

V': + 2 −

Max @ $x = 2$

$l = 12 - 4 = 8\,ft$

$w = 8\,ft$

$h = 2\,ft$

4. An open box (no lid) with a square base has a volume of 4 cubic feet. What dimensions will minimize the surface area?

$V = xxy$

$4 = x^2 y$

$y = \frac{4}{x^2}$

$SA = x^2 + 4xy$

$SA = x^2 + 4x\left(\frac{4}{x^2}\right)$

$SA = x^2 + \frac{16}{x}$

$SA = x^2 + 16x^{-1}$

$SA' = 2x - 16x^{-2}$

$0 = 2x - \frac{16}{x^2}$

$\frac{16}{x^2} = \frac{2x}{1}$

$\frac{\cancel{8}x^3}{\cancel{2}} = \frac{16}{2}$

$\sqrt[3]{x^3} = \sqrt[3]{8}$

$x = 2$

$x > 0$

$y > 0$

2ft x 2ft x 1ft

5. A sphere has a radius of 6 feet. What are the dimensions of the cylinder with the greatest volume that can be inscribed in the sphere? What is the maximum volume of this cylinder?