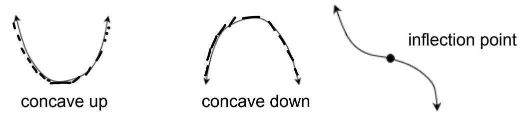
THE SECOND DERIVATIVE TEST

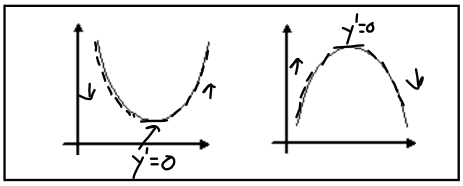
The first derivative describes the direction of the function. The second derivative describes the concavity of the original function. Concavity describes the direction of the curve, how it bends...



Just like direction, concavity of a curve can change, too. The points of change are called ***inflection points***.

Oct 31-10:38 AM

CONCAVITY EXPLORATION: Draw small tangent lines at points along the curves below. What do you notice about the slopes of the tangent lines (the derivatives) as you move from left to right at these points?



Dec 17-1:54 PM

TEST FOR CONCAVITY

If $f''(x)>0$, then graph of *f* is concave up.

If $f''(x)<0$, then graph of *f* is concave down.

SUMMARY OF FIRST AND SECOND DERIVATIVE TESTS

	$f'(x)>0$	$f'(x)<0$	$f'(x)=0$
$f''(x)>0$	Increasing Concave Up	Decreasing Concave Up	Relative Minimum Concave Up
$f''(x)<0$	Increasing Concave Down	Decreasing Concave Down	Relative Minimum Concave Down
$f''(x)=0$	Increasing Inflection Point	Decreasing Inflection Point	Function is "smooth, level" possible inflection point

Aug 6-5:25 PM

EX #1: Given $f(x)=\frac{1}{3}x^3-x$, determine the open intervals on which the graph is concave upward or downward.

$f'(x)=x^2-1$

$f''(x)=2x$

$2x=0$

$x=0$

$\sqrt{x^2}=\sqrt{1}$

$x=\pm 1$

concave down $-$ | concave up $+$

$f''(x)$ ← 0 →

$f'(x)$: $+$ | -1 | $-$ | 1 | $+$

Increases: $(-\infty,-1)\cup(1,\infty)$

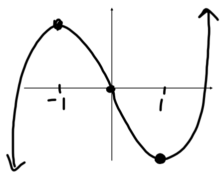
Decreases: $(-1,1)$

Oct 31-11:17 AM

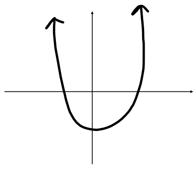
EX #2: Graphs and Derivatives

The concavity $(f''(x))$ and direction $(f'(x))$ of the function $(f(x))$ is related to the slope of the derivative.

$f(x)=\frac{1}{3}x^3-x$



$f'(x)=x^2-1$



$f'(x)$ $+$ -1 $-$ 1 $+$

Summary

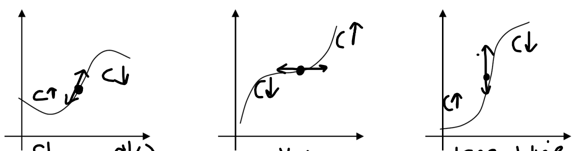
$f''(x)$ $-$ 0 $+$

Oct 31-11:31 AM

POINTS OF INFLECTION:

The concavity of *f* changes at a point of inflection.

Where $f''(x)=0$ or $f''(x)=\textit{does not exist}$



$f'>0$ $f''(x)=0$

$f''(x)=0$

tangent line
$f''(x)$ dne

Oct 31-11:43 AM

EX #3: Determine any points of inflection and discuss concavity of the graph of $f(x)=x^4-4x^3$

$f'(x)=4x^3-12x^2$

Test # $f''(x)$	+		−		+
Critical #		0		2	
Sign $f''(x)=$		possible POI		possible POI	

$f''(x)=12x^2-24x$

$12x^2-24x=0$

$12x(x-2)=0$

$x=0 \mid x=2$

Concave up $(-\infty,0)\cup(2,\infty)$

Concave down $(0,2)$

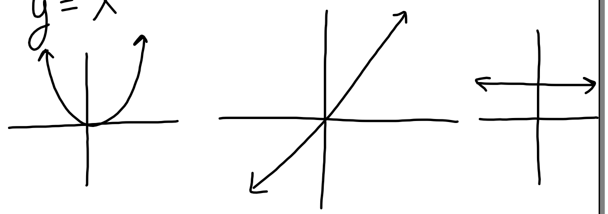
POI $(0,0)$, $(2,-16)$

Aug 6-6:15 PM

Function	Derivative
① Max/Min points	① Cross x-axis
② Inflection points	② Max/Min points
③ increasing	③ above x-axis
④ decreasing	④ below x-axis

Jan 7-10:03 AM

$y=x^2$ $\quad y'=2x$ $\quad y''=2$



Jan 7-10:09 AM