

Do Now

The Quotient Rule

$$\frac{d}{dx} \left[\frac{f(x)}{g(x)} \right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

Find an equation of the tangent line to the graph of f at the point $(-8, 5)$

$$f(x) = \frac{x}{2x+4}$$

$$\frac{2x+4(1) - x(2)}{(2x+4)^2} \rightarrow \frac{2x+4-2x}{(2x+4)^2} \rightarrow \frac{4}{(2x+4)^2}$$

$$y - 5 = \frac{4}{144}(x + 8)$$

$$\frac{4}{(2(-8)+4)^2} \rightarrow \frac{4}{(-12)^2} \rightarrow \frac{4}{144}$$

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Given the differentiable functions, $f(x)$ and $g(x)$. Using the table below,

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
6	-3	7	2	-4

A.) Find the equation of the line tangent to $g(x)$ at $x = 6$.

$$m = -4 \quad (6, 7) \quad y - 7 = -4(x - 6)$$

B.) Find the equation of the line tangent to $f(x)$ at $x = 6$.

$$(6, -3) \quad m = 2 \quad y + 3 = 2(x - 6)$$

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EX #6: Using the Constant Multiple Rule to Rewrite

Function	Rewrite	Differentiate	Simplify
$y = \frac{x^2 - 4x}{8}$	$\frac{1}{8}x^2 - \frac{1}{2}x$	$\frac{1}{4}x - \frac{1}{2}$	$\frac{x}{4} - \frac{1}{2} \rightarrow \frac{x}{4} - \frac{2}{4}$ $\frac{x-2}{4}$
$y = \frac{3x^3}{5}$	$\frac{3}{5}x^3$	$\frac{9}{5}x^2$	$\frac{9x^2}{5}$
$y = \frac{6x^{\frac{5}{2}}}{x}$	$6x^{\frac{3}{2}}$	$9x^{\frac{1}{2}}$	$9\sqrt{x}$
$y = \frac{6x^4 + x^3 - 2x}{\sqrt{x}}$	$6x^{\frac{7}{2}} + x^{\frac{5}{2}} - 2x^{\frac{1}{2}}$	$21x^{\frac{5}{2}} + \frac{5}{2}x^{\frac{3}{2}} - \frac{1}{2}x^{-\frac{1}{2}}$	$21\sqrt{x^5} + \frac{5}{2}\sqrt{x^3} - \frac{1}{\sqrt{x}}$

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Higher Order Derivatives

Did you realize that it is possible to take the derivative of a derivative? We will need this ability in the study of particle motion and objects in motion, especially related to physics in the near future.

$s(t)$	Position function
$v(t) = s'(t)$	Velocity function
$a(t) = v'(t) = s''(t)$	Acceleration function

First derivative:	y'	$f'(x)$	$\frac{dy}{dx}$	$\frac{d}{dx}[f(x)]$	$D_x[y]$
Second derivative:	y''	$f''(x)$	$\frac{d^2y}{dx^2}$	$\frac{d^2}{dx^2}[f(x)]$	$D_x^2[y]$
Third derivative:	y'''	$f'''(x)$	$\frac{d^3y}{dx^3}$	$\frac{d^3}{dx^3}[f(x)]$	$D_x^3[y]$
Fourth derivative:	$y^{(4)}$	$f^{(4)}(x)$	$\frac{d^4y}{dx^4}$	$\frac{d^4}{dx^4}[f(x)]$	$D_x^4[y]$

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EX #7: For each of the following, find the second derivative.

A.) $f(x) = \frac{3}{2}x^3 + 5x^2 - 6x + 1$

$$f'(x) = \frac{9}{2}x^2 + 10x - 6$$

$$f''(x) = 9x + 10$$

B.) $g(x) = \frac{x^2 - 4x - 5}{x}$

$$\left. \begin{aligned} g(x) &= x - 4 - 5x^{-1} \\ g'(x) &= 1 + 5x^{-2} \\ g''(x) &= -10x^{-3} \end{aligned} \right\} \begin{aligned} &\frac{x(2x-4) - (x^2-4x-5)(1)}{x^2} \\ &\frac{2x^2-4x-x^2+4x+5}{x^2} \\ &\frac{x^2+5}{x^2} \rightarrow \frac{x^2(2x) - (x^2+5)(1)}{x^4} \end{aligned}$$

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C.) $y = \frac{x}{x+2}$

$$y' = \frac{(x+2)(1) - x(1)}{(x+2)^2}$$

$$y' = \frac{x+2-x}{(x+2)^2} \rightarrow \frac{2}{(x+2)^2}$$

We haven't learned chain rule yet

$$\frac{2}{x^2+4x+4} \rightarrow y'' = \frac{(x+4x+4)(2) - 2(2x+4)}{(x+2)^4}$$

D.) $f(x) = 4\sqrt{x} - \frac{6}{\sqrt{x}}$

$$f(x) = 4x^{\frac{1}{2}} - 6x^{-\frac{1}{2}}$$

$$f'(x) = 2x^{-\frac{1}{2}} + 3x^{-\frac{3}{2}}$$

$$f''(x) = -x^{-\frac{3}{2}} - \frac{9}{2}x^{-\frac{5}{2}}$$

$$= -\frac{1}{\sqrt{x^3}} - \frac{9}{2\sqrt{x^5}}$$

$$y'' = \frac{-2(2x+4)}{(x+2)^4} \rightarrow \frac{-4(x+2)}{(x+2)^4} \rightarrow \frac{-4}{(x+2)^3}$$

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