

10-22-19

Aim: The Chain Rule

Do Now

Use the table to write the equation for tangent lines at given values of x .

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
-1	-2	4	0	3
2	-5	0	-3	DNE

- 1) Tangent of
- $f(x)$
- at
- $x = 2$

$$(2, -5) \quad m = -3$$

$$y + 5 = -3(x - 2)$$

- 2) Tangent of
- $g(x)$
- at
- $x = -1$

$$(-1, 4) \quad m = 3$$

$$y - 4 = 3(x + 1)$$

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The Chain Rule:

If $y = f(u)$ is a differentiable function of u , and $u = g(x)$ is a differentiable function of x , then $y = f(g(x))$ is a differentiable function of x and

$$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$$

$$\text{Or } f'(g(x)) \cdot g'(x)$$

This looks more complicated than it really is. Basically, the Chain Rule says to **MULTIPLY THE DERIVATIVE OF THE INSIDE FUNCTION BY THE DERIVATIVE OF THE OUTSIDE FUNCTION.**

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EX #1: Find the derivative of $f(x) = (2x+3)^2$
with and without the chain rule.

A.) without chain rule

$$\begin{aligned}
 f(x) &= (2x+3)^2 \\
 f(x) &= \underbrace{(2x+3)}_u \underbrace{(2x+3)}_{u'} \\
 f'(x) &= \underbrace{(2x+3)}_u (2) + \underbrace{(2x+3)}_{u'} (2) \\
 f'(x) &= 4x+6+4x+6 \\
 &= 8x+12 = 4(2x+3)
 \end{aligned}$$

B.) with chain rule

$$\begin{aligned}
 f(x) &= (2x+3)^2 \\
 u &= 2x+3 \\
 u^2 \\
 2u \cdot u' \\
 2u(2) \\
 2(2x+3)(2) &\rightarrow 4(2x+3)
 \end{aligned}$$

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$$\begin{aligned}
 y &= (5x+3)^4 \\
 y' &= 4(5x+3)^3(5) \\
 &= 20(5x+3)^3
 \end{aligned}$$

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The Chain Rule: $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ or $f' g(x) g'(x)$

(Derivative of the outside function) x (derivative of the inside function)

EX #2: Find $f'(x)$ given $f(x) = (4 - x^2)^3$

The inside function is $4 - x^2$

The outside function is u^3

$$f'(x) = 3(4 - x^2)^2 (-2x)$$

$$= -6x(4 - x^2)^2$$

$$u = 4 - x^2$$

$$f'(x) = 3u^2 \cdot (-2x)$$

$$= 3(4 - x^2)^2 (-2x)$$

$$= -6x(4 - x^2)^2$$

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For each function below, find the derivative.

EX #3: $f(x) = \sqrt{(x^2 - 1)^3}$

$$f(x) = (x^2 - 1)^{3/2}$$

$$f'(x) = \frac{3}{2}(x^2 - 1)^{1/2} \cdot (2x)$$

$$= 3x\sqrt{x^2 - 1}$$

EX #4: $f(x) = \frac{-7}{(2x - 3)^2}$

$$f(x) = -7(2x - 3)^{-2}$$

$$f'(x) = 14(2x - 3)^{-3} (2)$$

$$= 28(2x - 3)^{-3}$$

$$= \frac{28}{(2x - 3)^3}$$

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EX #5: $y = \frac{1}{2x-3}$

$$y = (2x-3)^{-1}$$

$$\frac{dy}{dx} = -(2x-3)^{-2} (2)$$

$$= \frac{-2}{(2x-3)^2}$$

EX #6: $y = (5-4x^2)^{2/3}$

$$\frac{dy}{dx} = \frac{2}{3} (5-4x^2)^{-1/3} (-8x)$$

$$= \frac{-16x}{3 \sqrt[3]{5-4x^2}}$$

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EX #7: $y = \frac{-2}{\sqrt[3]{6x+3}}$

$$y = -2 (6x+3)^{-1/3}$$

$$y' = \frac{2}{3} (6x+3)^{-4/3} (6)$$

$$y' = \frac{4}{\sqrt[3]{(6x+3)^4}}$$

EX #8: $y = -3\sqrt{x^2-3x-4}$

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