

Name _____ Calculus
Review

This review sheet should NOT serve as your only review. You should review all notes and tests.

Questions 1 through 7 refer to the graph of $y = f(x)$ shown to the right.

1. $\lim_{x \rightarrow 1} f(x) = -\infty$
2. $\lim_{x \rightarrow 1} f(x) = \infty$
3. $\lim_{x \rightarrow 1} f(x) = \text{DNE}$
4. $\lim_{x \rightarrow -3} f(x) = 1$
5. $\lim_{x \rightarrow 3} f(x) = -1$
6. $\lim_{x \rightarrow \infty} f(x) = \infty$
7. $\lim_{x \rightarrow -\infty} f(x) = -2$

For each of the following functions, use the definition of derivative to find $f'(x)$.
Recall: $f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$

a) $f(x) = 2x^2 - 8x + 5$

$$f'(x) = \lim_{h \rightarrow 0} \frac{2(x+h)^2 - 8(x+h) + 5 - (2x^2 - 8x + 5)}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2(x^2 + 2xh + h^2) - 8x - 8h + 5 - 2x^2 + 8x - 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - 8x - 8h + 5 - 2x^2 + 8x - 5}{h}$$

$$= \lim_{h \rightarrow 0} \frac{2xh + 2h^2 - 8h}{h}$$

$$= \lim_{h \rightarrow 0} (2x + 2h - 8)$$

$$= 2x - 8$$

b) $f(x) = \sqrt{x+2}$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{x+h+2} - \sqrt{x+2}}{h}$$

$$= \lim_{h \rightarrow 0} \frac{\sqrt{x+h+2} + \sqrt{x+2}}{\sqrt{x+h+2} + \sqrt{x+2}} \cdot \frac{\sqrt{x+h+2} - \sqrt{x+2}}{\sqrt{x+h+2} - \sqrt{x+2}}$$

$$= \lim_{h \rightarrow 0} \frac{(x+h+2) - (x+2)}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{h}{h(\sqrt{x+h+2} + \sqrt{x+2})}$$

$$= \lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h+2} + \sqrt{x+2}}$$

$$= \frac{1}{2\sqrt{x+2}}$$

Find the derivative of each of the following:

9. $f(x) = 8x + 2\sqrt[3]{x} - \frac{3}{x^2}$
 $f'(x) = 8x^2 + 2\sqrt[3]{x^2} - 3x^{-3}$
 $f'(x) = 8 + \frac{2}{3}x^{-\frac{1}{3}} - 3x^{-4}$

10. $f(x) = \sin(5x^3 + 2x)$
 $f'(x) = \cos(5x^3 + 2x)(15x^2 + 2)$
 $f'(x) = \frac{1}{5}(5x^2 + 2x)(10x^2 + 2)$

11. $f(x) = \sqrt[3]{(5x^2 + 2x)^2}$
 $f'(x) = \frac{2}{3}(5x^2 + 2x)^{\frac{1}{3}}(10x^2 + 2)$

12. Find the slope of the line tangent to $y = x \cos(x)$ when $x = 0$.

$$y' = x(-\sin(x)) + \cos(x)(1)$$

$$= 0(-\sin(0)) + \cos(0)(1)$$

$$= 1$$

13. Write the equation of the line tangent to $y = 3x^2 - 2x + 1$ when $x = -1$.

$$y = 3(-1)^2 - 2(-1) + 1$$

$$= 3 + 2 + 1 = 6$$

$$y' = 6x - 2 \rightarrow 6(-1) - 2 \rightarrow -8$$

14. The following table records the values of f , f' , g , and g' at $x = 1$, $x = 2$, and $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

If $n(x) = \frac{f(x)}{g(x)}$, $(x) = f(g(x))$, find the value of each of the following:

a) $n'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{(g(2))^2} \rightarrow \frac{3(4) - (-5)(4)}{3^2} \rightarrow \frac{28}{9}$

b) $h(x) = f(g(x))$
 $h'(x) = f'(g(x)) \cdot g'(x)$
 $f(x) = (x^2 - 2x - 1)^{-\frac{1}{3}}$
 $f'(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{2}{3}}(2x - 2)$
 $f'(0) = \frac{2}{3}(0^2 - 2(0) - 1)^{-\frac{2}{3}}(2(0) - 2) = 4 \cdot \frac{2}{3} = \frac{8}{3}$
 $= \frac{2}{3}(-1)^{-\frac{2}{3}} \cdot (-2) \rightarrow -\frac{4}{3}$

15. If $f(x) = \sqrt[3]{(x^2 - 2x - 1)^2}$, then $f'(0) = ?$

$$f'(x) = \frac{2}{3}(x^2 - 2x - 1)^{-\frac{1}{3}}(2x - 2)$$

$$= \frac{2}{3}(0^2 - 2(0) - 1)^{-\frac{1}{3}}(2(0) - 2) = \frac{2}{3}(-1)^{-\frac{1}{3}} \cdot (-2) \rightarrow -\frac{4}{3}$$

16. Is $h(x)$ continuous for all real numbers? If so show why.

$$h(x) = \begin{cases} x+3, & x \leq -2 \\ -x^2, & x > -2 \end{cases}$$

$$h(-2) = 1$$

$$\lim_{x \rightarrow -2} h(x) \neq h(-2)$$

$$\lim_{x \rightarrow -2^-} = 1$$

$$\lim_{x \rightarrow -2^+} = -4$$

DNE $\neq 1$ $\therefore h(x)$ is not continuous

17. Evaluate $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

$$2+5 = \boxed{7}$$

18. Evaluate $\lim_{x \rightarrow \infty} \frac{2x^3 - 3}{3x^3 + 25}$ = $\boxed{\frac{2}{3}}$

19. Find the derivative of the following:

a) $f(x) = e^{2x} \sin(3x)$

$$\begin{aligned} f'(x) &= e^{2x} ((\cos(3x)(3) + \sin(3x))e^{2x} \cdot 2) \\ &= 3e^{2x} \cos(3x) + 2e^{2x} \sin(3x) \end{aligned}$$

b) $y = \frac{\ln(2x)}{\sqrt{x^2 + 5x}}$

$$y' = \underline{\ln(2x) \cdot 2} - \underline{\ln(2x) \frac{1}{2} (x^2 + 5x)^{-\frac{1}{2}} (2x)}$$

$$y' = \frac{1}{x} \sqrt{x^2 + 5x} - \frac{x^2 + 5x}{x \sqrt{x^2 + 5x}} \cdot \underline{\ln(2x) (x^2 + 5x)^{-\frac{1}{2}}}$$