Differentiability and Continuity

The alternative limit form is useful in investigating the relationship between differentiability and continuity. The derivative of f at c (provided the limit exists) is . . .

$$f'(c)=\lim_{x\to c}\frac{f(x)-f(c)}{x-c}$$

The existence of the limit in this alternate form requires that :

☆ 1. The one-sided limits exist, and

☆ 2. They are equal.

We say that f is differentiable on the closed interval $[a, b]$ if it is differentiable on (a, b) and if the derivative from the right at a, and the derivative from the left at b, both exist.

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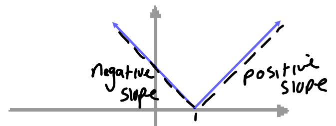
EX #1: Is the function $g(x)=|x-1|$ differentiable at $x = 1$?

$\lim_{x\to 1^-} = 0$

$\lim_{x\to 1^+} = 0$

$\lim_{x\to 1} = 0$ limit is continuous

$f(1)=0$



The function is continuous everywhere but not differentiable at a sharp corner

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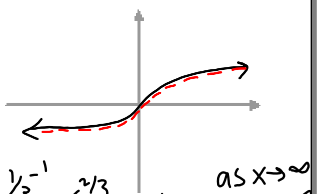
EX #2: A look at vertical tangent lines and using the alternate form of the derivative

Given: $f(x)=\sqrt[3]{x}$

$\lim_{x\to 0}\dfrac{\sqrt[3]{x}-\sqrt[3]{0}}{x-0}$

$\dfrac{\sqrt[3]{x}}{x}$



$\dfrac{x^{1/3}}{x} \to x^{1/3-1} \to x^{-2/3} \to \dfrac{1}{x^{2/3}}$ as $x\to\infty$ as $x\to -\infty$ 0

look at behavior of the tangent line when $x=0$

This function is continuous & differentiable

Sep 18-9:01 AM

Differentiability Implies Continuity:

If f is differentiable at $x = c$, then f is continuous at $x = c$.

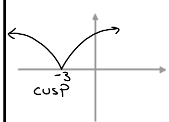
1. If a function is differentiable at $x = c$,then it is continuous at $x = c$.
   So, differentiability implies continuity.

2. It is possible for a function to be continuous at $x = c$ and not be differentiable at $x = c$.
   So, continuity does not imply differentiability.

SHARP CORNER, CUSP, or VERTICAL TANGENT LINE

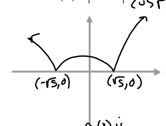
EX #3: Describe the x-values at which f is differentiable.

A.) $f(x)=(x+3)^{\frac{2}{3}}$



$f(x)$ is continuous everywhere, but not differentiable at $x=-3$

B.) $g(x)=|x^2-5|$



$g(x)$ is continuous everywhere, but differentiable at $x=\pm\sqrt{5}$

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EX #4: Find the derivatives from the left and from the right at $x = 1$ (if they exist).
Is the function differentiable at $x = 1$?

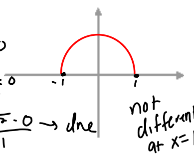
Given: $f(x)=\sqrt{1-x^2}$

$\sqrt{1-x^2}=0$

$1-x^2=0$

$(1-x)(1+x)=0$

$x=\pm 1$



$\lim_{x\to 1^+}\dfrac{\sqrt{1-x^2}-\sqrt{1-1}}{x-1} \to \dfrac{\sqrt{1-x^2}-0}{x-1} \to$ dne

not differentiable at $x=1$

$\lim_{x\to 1^-}\dfrac{\sqrt{1-x^2}-0}{x-1} \to -x^n$

EX #5: Find the value(s) of the constant a where the function is continuous over all reals.

$$f(x)=\begin{cases} ax^2-x & ;x<3 \\ 2a & ;\ x\geq 3\end{cases}$$

$ax^2-x=2a$

$(3)^2a-3=2a$

$9a-3=2a$

$-3=-7a$

$a=3/7$

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EX #6: Show algebraically, using the alternate form of derivative, why $g(x)$ is not differentiable.

$$g(x)=\begin{cases} x^2; x\leq 1 \\ x; x>1\end{cases}$$

$\lim_{x\to 1^-}\dfrac{f(x)-f(c)}{x-c}$

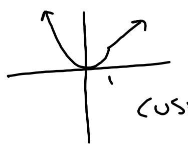
$\dfrac{x^2-1}{x-1} \to \dfrac{(x+1)\cancel{(x-1)}}{\cancel{x-1}} \to x+1 \to \boxed{2}$

$\lim_{x\to 1^+}\dfrac{f(x)-f(c)}{x-c}$

$\dfrac{x-1}{x-1} \to \boxed{1}$

$\lim_{x\to 1^-} \neq \lim_{x\to 1^+}$

$g(x)$ is not differentiable at $x=1$



cusp at $x=1$

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