

11/12/19

Aim: Implicit Differentiation

Implicit Differentiation

We have been able to differentiate functions that are solved for y explicitly up to this point. Now we want to consider functions of the type $x^2 - 2y^2 + 4y = 2$. You can see that it would be quite challenging to solve for y as a function of x , explicitly.

Implicit Differentiation

- Realize differentiation is taking place with respect to x .
- When you differentiate terms involving x alone, you can differentiate as usual.
- When you differentiate terms involving y , you must apply the Chain Rule (because you are assuming that y is defined implicitly as a differentiable function of x).

Nov 12-8:06 AM

Find $\frac{dy}{dx}$ for $x^2 - y^2 = 16$ at $(-5, 3)$

$$2x - 2y \frac{dy}{dx} = 0$$

$$\frac{-2y \frac{dy}{dx}}{-2y} = \frac{-2x}{-2y}$$

$$\frac{dy}{dx} = \frac{x}{y} = \frac{-5}{3}$$

Nov 12-8:09 AM

Find $\frac{dy}{dx}$ for $xy + y = 8$ at $(3, 2)$

$u = x$
 $u' = 1$
 $v = y$
 $v' = \frac{dy}{dx}$

$$x \frac{dy}{dx} + y(1) + \frac{dy}{dx} = 0$$

$$\frac{\frac{dy}{dx}(x+y)}{x+y} = \frac{-y}{x+y} = \frac{-2}{4} = -\frac{1}{2}$$

Nov 12-8:18 AM

Find the instantaneous rate of change at $(1, 1)$ for $x + 3xy - 2y^2 = 2$

$u = 3x$
 $u' = 3$
 $v = y$
 $v' = \frac{dy}{dx}$

$$1 + 3x(\frac{dy}{dx}) + y(3) - 4y \frac{dy}{dx} = 0$$

$$3x \frac{dy}{dx} - 4y \frac{dy}{dx} = -1 - 3y$$

$$\frac{\frac{dy}{dx}(3x - 4y)}{3x - 4y} = \frac{-1 - 3y}{3x - 4y}$$

$$\frac{dy}{dx} = \frac{-1 - 3(1)}{3(1) - 4(1)} = \frac{-4}{-1} \rightarrow 4$$

Nov 12-8:18 AM

Find $\frac{dy}{dx}$ given that $y^3 + 5y^2 - 5y - x^2 = -4$

$$3y^2 \frac{dy}{dx} + 10y \frac{dy}{dx} - 5 \frac{dy}{dx} - 2x = 0$$

$$\frac{\frac{dy}{dx}(3y^2 + 10y - 5)}{3y^2 + 10y - 5} = \frac{2x}{3y^2 + 10y - 5}$$

Nov 12-8:19 AM

Find $\frac{dy}{dx}$ for $x + \sqrt{xy} = 6$ at $(3, 3)$

$$x + (xy)^{\frac{1}{2}} = 6$$

$$1 + \frac{1}{2}(xy)^{-\frac{1}{2}}(x \frac{dy}{dx} + y(1)) = 0$$

$$1 + \frac{1}{2\sqrt{xy}}(x \frac{dy}{dx} + y) = 0$$

$$1 + \frac{1}{2\sqrt{3 \cdot 3}}(3 \frac{dy}{dx} + 3) = 0$$

$$1 + \frac{1}{6}(3 \frac{dy}{dx} + 3) = 0$$

$$1 + \frac{1}{2} \frac{dy}{dx} + \frac{1}{2} = 0$$

$$\frac{3}{2} + \frac{1}{2} \frac{dy}{dx} = 0$$

$$2(\frac{1}{2} \frac{dy}{dx}) = (-\frac{3}{2}) \cdot 2$$

$$\frac{dy}{dx} = -3$$

Nov 12-8:19 AM