Find points where there are horizontal tangent lines

$x^2+xy+y^2=4$

$2x+x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}(x+2y)=-2x-y$

$\frac{dy}{dx}=\frac{-2x-y}{x+2y}$

$-2x-y=0$

$y=-2x$

$x^2+x(-2x)+(-2x)^2=4$

$x^2-2x^2+4x^2=4$

$3x^2=4$

$x=\pm\sqrt{\frac{4}{3}}=\pm\frac{2}{\sqrt{3}}$

$y=-2x$

$=-2\left(\frac{2}{\sqrt{3}}\right)$

$=-\frac{4}{\sqrt{3}}$

$\left(\frac{2}{\sqrt{3}},-\frac{4}{\sqrt{3}}\right)$

$\left(-\frac{2}{\sqrt{3}},\frac{4}{\sqrt{3}}\right)$

$y=-2\left(-\frac{2}{\sqrt{3}}\right)$

$y=\frac{4}{\sqrt{3}}$

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When is the slope undefined?

$\frac{dy}{dx}=\frac{-2x-y}{x+2y}$

Set denom = 0

$\frac{x}{-2}=y$

$x+2y=0$

$\frac{x}{-2}=\frac{-2y}{-2}$

$y=\frac{-x}{2}$

$x^2+xy+y^2=4$

$x^2+x\left(-\frac{x}{2}\right)+\left(-\frac{x}{2}\right)^2=4$

$4\left(\frac{x^2}{1}-\frac{x^2}{2}+\frac{x^2}{4}=4\right)$

$4x^2-2x^2+x^2=16$

$\frac{3x^2}{3}=\frac{16}{3}$

$x=\pm\sqrt{16/3}=\pm\frac{4}{\sqrt{3}}$

$\left(\frac{4}{\sqrt{3}},-\frac{2}{\sqrt{3}}\right)$

$\left(-\frac{4}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$

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$x+2y=0$

$\frac{2y}{-1}=\frac{-x}{-1}$

$x=-2y$

$x^2+xy+y^2=4$

$(-2y)^2+(-2y)(y)+y^2=4$

$4y^2-2y^2+y^2=4$

$\frac{3y^2}{3}=\frac{4}{3}\rightarrow y=\pm\frac{2}{\sqrt{3}}$

$\left(\frac{-4}{\sqrt{3}},\frac{2}{\sqrt{3}}\right)$

$\left(\frac{4}{\sqrt{3}},\frac{-2}{\sqrt{3}}\right)$

$x=-2y$

$x=-2\left(\frac{2}{\sqrt{3}}\right)$

$x=-2\left(\frac{-2}{\sqrt{3}}\right)$

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Practice

Find the points of all horizontal and vertical tangents

1) $x^2+xy+y^2=9$

2) $x^2+y^2-4x+4y+8=16$

1) $2x+x\frac{dy}{dx}+y+2y\frac{dy}{dx}=0$

$\frac{dy}{dx}(x+2y)=-2x-y$

$\frac{dy}{dx}=\frac{-2x-y}{x+2y}$

HTL (Points)

$x^2+xy+y^2=9$

$x^2+x(-2x)+(-2x)^2=9$

$-2x-y=0$

$y=-2x$

$x^2-2x^2+4x^2=9$

$3x^2=9$

$x^2=3$

$x=\pm\sqrt{3}$

$(\sqrt{3},-2\sqrt{3})$

$(-\sqrt{3},2\sqrt{3})$

VTL (points)

$x+2y=0$

$x=-2y$

$x^2+xy+y^2=9$

$(-2y)^2+(-2y)(y)+y^2=9$

$4y^2-2y^2+y^2=9$

$3y^2=9$

$y^2=3$

$y=\pm\sqrt{3}$

$x=-2y$

$(2\sqrt{3},\sqrt{3})$

$(\sqrt{3},-\sqrt{3})$

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