

Find points where there are horizontal tangent lines  
 $x^2 + xy + y^2 = 4$


$2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$   
 $\frac{dy}{dx}(x+2y) = \frac{-2x-y}{x+2y}$

$-2x - y = 0$   
 $y = -2x$

$x^2 + x(-2x) + (-2x)^2 = 4$   
 $x^2 - 2x^2 + 4x^2 = 4$   
 $3x^2 = 4$   
 $x = \pm \frac{2}{\sqrt{3}}$

$y = -2x$   
 $y = -2(\frac{2}{\sqrt{3}}) = -\frac{4}{\sqrt{3}}$   
 $y = -2(-\frac{2}{\sqrt{3}}) = \frac{4}{\sqrt{3}}$

$(\frac{2}{\sqrt{3}}, -\frac{4}{\sqrt{3}})$   
 $(-\frac{2}{\sqrt{3}}, \frac{4}{\sqrt{3}})$



Nov 21-1:42 PM

When is the slope undefined?

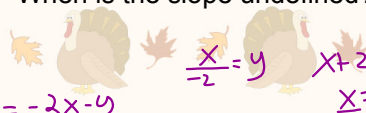
$x + 2y = 0$   
 $\frac{x}{-2} = y$   
 $\frac{x}{-2} = -\frac{2y}{-2}$   
 $x^2 + xy + y^2 = 4$

Set denom = 0  
 $\frac{dy}{dx} = \frac{-2x-y}{x+2y}$

$y = -\frac{x}{2}$

$x^2 + x(-\frac{x}{2}) + (-\frac{x}{2})^2 = 4$   
 $4(\frac{x^2}{1} - \frac{x^2}{2} + \frac{x^2}{4}) = 4$   
 $4x^2 - 2x^2 + x^2 = 16$   
 $3x^2 = 16$   
 $x = \pm \sqrt{\frac{16}{3}} = \pm \frac{4}{\sqrt{3}}$

$(\frac{4}{\sqrt{3}}, -\frac{2}{\sqrt{3}})$   
 $(-\frac{4}{\sqrt{3}}, \frac{2}{\sqrt{3}})$



Nov 21-1:45 PM

$x + 2y = 0$   
 $2y = -x$   
 $x = -2y$

$x^2 + xy + y^2 = 4$   
 $(-2y)^2 + (-2y)(y) + y^2 = 4$   
 $4y^2 - 2y^2 + y^2 = 4$   
 $3y^2 = 4$   
 $y = \pm \frac{2}{\sqrt{3}}$

$x = -2y$   
 $x = -2(\frac{2}{\sqrt{3}}) = -\frac{4}{\sqrt{3}}$   
 $x = -2(-\frac{2}{\sqrt{3}}) = \frac{4}{\sqrt{3}}$

$(-\frac{4}{\sqrt{3}}, \frac{2}{\sqrt{3}})$   
 $(\frac{4}{\sqrt{3}}, -\frac{2}{\sqrt{3}})$

Nov 22-9:53 AM

Practice  
 Find the points of all horizontal and vertical tangents

1)  $x^2 + xy + y^2 = 9$   
 2)  $x^2 + y^2 - 4x + 4y + 8 = 16$

1)  $2x + x \frac{dy}{dx} + y + 2y \frac{dy}{dx} = 0$   
 $\frac{dy}{dx}(x+2y) = \frac{-2x-y}{x+2y}$

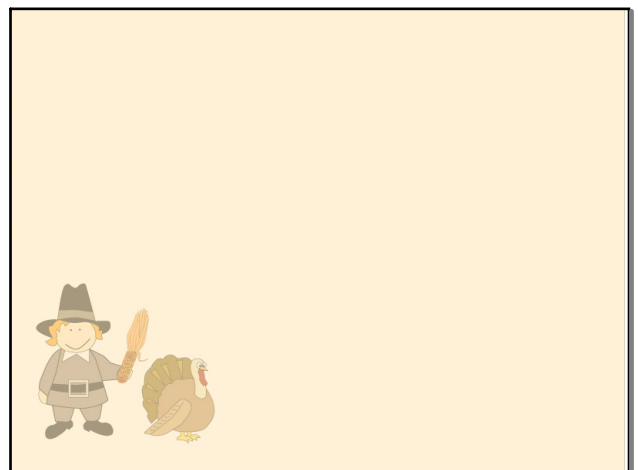
HTL (points)  
 $x^2 + xy + y^2 = 9$   
 $x^2 + x(-2x) + (-2x)^2 = 9$   
 $x^2 - 2x^2 + 4x^2 = 9$   
 $3x^2 = 9$   
 $x = \pm 3$   
 $(3, -2)$   
 $(-3, 2)$

VTL (points)  
 $x + 2y = 0$   
 $x = -2y$   
 $x^2 + xy + y^2 = 9$   
 $(-2y)^2 + (-2y)(y) + y^2 = 9$   
 $4y^2 - 2y^2 + y^2 = 9$   
 $3y^2 = 9$   
 $y = \pm 3$   
 $(2, 3)$   
 $(-2, -3)$

Nov 21-1:45 PM



Nov 21-1:45 PM



Nov 21-1:45 PM