11/4/19
Aim: What is Rectilinear Motion?

Do Now

Find the derivative of each function. Simplify, if possible.

1) $y = \tan^2 x$

2) $y = \sec^2(x - 1)$

3) $y = \frac{x^8}{\cos x}$

4) $y = \sqrt{\tan 3x}$

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POSITION, VELOCITY, and ACCELERATION

The derivative can determine slope and can also be used to determine the rate of change of one variable with respect to another. The function *s* that gives the position (relative to the origin) of an object as a function of time *t* is called a position function.

Position function:

The function *s*(*t*) or *x*(*t*) that gives the position, relative to the origin, of an object as a function of time *t*.

Sep 23-5:12 AM

RATES OF CHANGE

In our study of Calculus, we are often interested when an object (or particle) is speeding up, slowing down, stopped, or has no acceleration.

Recall the notation for the average rate of change (ARoC) of a function *y* = *f* (*x*) over an interval [x_0 , x_1]

Δy = change in *y*

Δx = change in *x*

REMEMBER the Average Rate of Change is the slope of the secant line.

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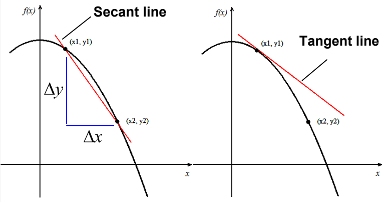
Average Velocity of an Object Over a Time Interval (A.R.O.C.)

Rate = $\frac{\text{Distance}}{\text{Time}}$

Average Velocity

$V_{avg} = \frac{\Delta s}{\Delta t} = \frac{\text{Change in distance}}{\text{Change in time}} = \frac{f(t+\Delta t) - f(t)}{\Delta t}$

The average velocity between t_1 and t_2 is the slope of the secant line,
and the instantaneous velocity at t_1 is the slope of the tangent line.



Jul 27-5:32 PM

Do Now Solutions

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HW Solutions

1. $y = \cos 4x$
$y' = -4 \sin 4x$

2. $y = \sin 2x - \cos 3x$
$y' = 2\cos 2x + 3\sin 3x$

3. $y = x \sin x$
$y' = x\cos x + \sin x$

6. $y = x^4 \sin^2 x$
$y' = x^4(2\sin x)\cos x + \sin^2 x(4x$
$\underline{\underline{y' = 2x^3 \sin x(x\cos x + 2\sin x)}}$

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Motion along a straight line.

Ex: A car travelling
A train .

Distance	Velocity	Acceleration
Position function	Rate of change of the distance	Rate of change of velocity
How far the object has traveled	Derivative of position	Direction
	Direction	Second derivative

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Instantaneous Velocity:

Tells how fast something is going at that exact instant and in which direction. That is, how the position is changing with respect to time represented by

$$\frac{ds}{dt} = s'(t) = v(t) = \lim_{\Delta t \to 0} \frac{s(t+\Delta t) - s(t)}{\Delta t}$$

Speed:

Tells how fast an object is going no matter which direction. Speed measures the rate at which the position changes.

$$\text{Speed} = |\text{velocity}|$$

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Acceleration:

Is the instantaneous rate of change of velocity. It tells how quickly the body speeds up or slows down; how fast the velocity is changing with respect to time.

$$s''(t) = \frac{d^2s}{dt^2} = v'(t) = \frac{dv}{dt} = a(t)$$

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TECHNIQUES FOR SPEEDING UP AND SLOWING DOWN:

- If velocity and acceleration have the ***same sign***, the object is ***speeding up***.
- If velocity and acceleration have ***opposite signs***, the object is ***slowing down***.

SPEEDING UP **SAME SIGNS**	**SLOWING DOWN** **OPPOSITE SIGNS**
Positive Velocity ●⟶ ●⟶ Positive Acceleration	Positive Velocity ●⟶ ⟵● Negative Acceleration
Negative Velocity ⟵● ⟵● Negative Acceleration	Negative Velocity ⟵● ●⟶ Positive Acceleration

Sep 23-5:23 AM

A particle moves along a horizontal line. Its position is $s(t) = t^3 - t^2 - 56t$, where t represents time.

a) What is the function for velocity?

b) What is the function for acceleration

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s(t)=position
v(t)=s'(t)=velocity
a(t)=v'(t)=s"(t)=acceleration

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1) Suppose the position of a particle on the x-axis is represented by $s(t)=t^3-6t^2+9t+5$ where $t \geq 0$. What is the position at time: $t=0$, $t=1$, $t=2$, $t=3$?

$s(0)=0^3-6(0)^2+9(0)+5=5$

$s(1)=1^3-6(1)^2+9(1)+5=9$

$s(2)=2^3-6(2)^2+9(2)+5=7$

$s(3)=3^3-6(3)^2+9(3)+5=5$

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$s(0)=0^3-6(0)^2+9(0)+5=5$

$s(1)=1^3-6(1)^2+9(1)+5=9$

$s(2)=2^3-6(2)^2+9(2)+5=7$

$s(3)=3^3-6(3)^2+9(3)+5=5$

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Velocity is the derivative of position. To find the distance traveled we can use velocity.

How do we know when the particle changed direction?

When velocity=0

What is the velocity function?

v(t)=s'(t)

$$v(t)=3t^2-12t+9$$

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When is the velocity=0?

$$0=3t^2-12t+9$$

Solve for t

The Particle is stopped at:

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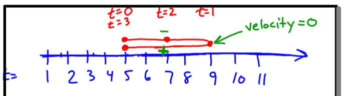

How do we find the distance traveled?

We evaluate the position function on intervals in which the velocity has the same sign.

From $t=0$ to $t=1$

$s(0)=5$

$s(1)=9$ Distance travelled = 4

From $t=1$ to $t=3$

$s(1)=9$

$s(3)=5$ Distance travelled = 4

Total Distance = 4+4=8

Total Distance

velocity=0

$$|s(0)-s(1)| + |s(1)-s(3)|$$

$$4 + 4 = \boxed{8}$$

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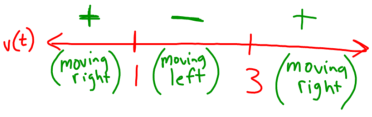
$s(t)=t^3-6t^2+9t+5$

$V(t)=3t^2-12t+9$

$V(t)=0=3(t-1)(t-3)$

$V=0$ @ $t=1$ $t=3$



velocity is (+) moving right

" is (−) " left

Right: $[0,1)\cup(3,\infty)$

Left: $(1,3)$

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$v(t) = 3t^2 - 12t + 9$

$a(t) = v'(t) = 6t - 12$

When is the particle not accelerating?

$a(t) = 0$

$0 = 6t - 12$

$12 = 6t$

$2 = t$

acceleration $= 0$ @ $t=2$

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① $s(t) = t^3 - 9t^2 + 15t + 6$

a) $t = 3$

$s(3) = (3)^3 - 9(3)^2 + 15(3) + 6$

$\boxed{-3}$

b) $s'(t) = v(t) = 3t^2 - 18t + 15$

$v(3) = 3(3)^2 - 18(3) + 15$

$= -12$

c) $s''(t) = v'(t) = a(t) = 6t - 18$

$a(3) = 6(3) - 18 = 0$

d) Moving at a constant speed

e) $v(t) = 0$

$3t^2 - 18t + 15 = 0$

$3(t^2 - 6t + 5) = 0$

$3(t-5)(t-1) = 0$

$t = 5$ | $t = 1$

f. + − + $t \geq 0$

$v(t)$ ←—1—5—→

Right $[0,1) \cup (5, \infty)$

left $(1,5)$

g. total distance traveled (first 4 sec)

$|s(0) - s(1)| + |s(4) - s(1)|$

$s(0) = 6$

$s(1) = 13$

$s(4) = -14$

$|6 - 13| + |-14 - 13|$

$7 + 27 = 34$

Nov 6-9:40 AM

$s(t) = 2t^3 - 9t^2 + 12t - 7$

a) $s(3) = 2$

b) $v(t) = 6t^2 - 18t + 12$

$v(3) = 12$

c) $a(t) = 12t - 18$

$a(3) = 18$

d) Speeding up b/c both $a(3)$ & $v(3)$ have the Same sign (Both +)

e) $6t^2 - 18t + 12 = 0$

$6(t^2 - 3t + 2) = 0$

$6(t-2)(t-1) = 0$

$t = 2$ | $t = 1$

f) $v(t)$ + —1— − —2— +

$[0,1) \cup (2, \infty)$

g) $|s(1) - s(0)| + |s(2) - s(1)| + |s(4) - s(2)|$

$s(0) = -7$

$s(1) = -2$

$s(2) = -3$

$s(4) = 25$

$|-2 \overset{+}{\ominus} 7| + |-3 \ominus 2| + |25 - -3|$

$5 + 1 + 28 = 34$

Nov 6-10:00 AM