

If $xy^2 - 5 - y^2 = x$ find the equation of the tangent line when $x=3$ in the fourth quadrant.

$$3y^2 - 5 - y^2 = 3$$

$$2y^2 = 8$$

$$\sqrt{y^2 = 4}$$

$$y = \pm 2$$

$(3, -2), (3, 2)$

$$y + 2 = \frac{3}{8}(x - 3)$$

$$x^2 y \frac{dy}{dx} + y^2 - 2y \frac{dy}{dx} = 1$$

$$2xy \frac{dy}{dx} - 2y \frac{dy}{dx} = 1 - y^2$$

$$\frac{dy}{dx} (2xy - 2y) = 1 - y^2$$

$$\frac{1 - (-2)^2}{2(3)(-2) - 2(-2)} = \frac{-3}{-8} = \frac{3}{8}$$

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$-5 - x^2 y - x^2 = 5y$

Find the coordinate point on the curve where $x = -2$ and the line tangent to the curve is horizontal.

$$0 - x^2 \frac{dy}{dx} - 2xy - 2x = 5 \frac{dy}{dx}$$

$$-x^2 \frac{dy}{dx} - 5 \frac{dy}{dx} = 2xy + 2x$$

$$\frac{dy}{dx} (-x^2 - 5) = \frac{2xy + 2x}{-x^2 - 5}$$

$$2xy + 2x = 0 \quad (-2, -1)$$

$$2(-2)y + 2(-2) = 0$$

$$-4y - 4 = 0$$

$$-4y = 4$$

$$y = -1$$

plug $(-2, -1)$ into orig equation, check if point exists on the curve

$$-5 = -5 \quad \checkmark$$

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Given the equation of the curve $x^2 + y^2 = 2x$, find all points on the curve where the line tangent is vertical.

$$2x + 2y \frac{dy}{dx} = 2$$

$$2y \frac{dy}{dx} = \frac{2 - 2x}{2y} = \frac{1 - x}{y}$$

$$y = 0$$

$$x^2 + 0 = 2x$$

$$x^2 - 2x = 0$$

$$x(x - 2) = 0$$

$$0, 2$$

$$(0, 0), (2, 0)$$

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$x = xy - y^3 + 4$

find equations of all tangent lines when $x = 4$

$(4, 0), (4, 2), (4, -2)$

$$4 = 4y - y^3 + 4$$

$$0 = 4y - y^3$$

$$0 = y(4 - y^2)$$

$$0 = y(2 - y)(2 + y)$$

$$y = 0 \quad y = 2 \quad y = -2$$

$$1 = x \frac{dy}{dx} + y - 3y^2 \frac{dy}{dx}$$

$$\frac{1 - y}{x - 3y^2} = \frac{dy}{dx} \frac{(x - 3y^2)}{x - 3y^2}$$

$$\frac{1 - 0}{4 - 3(0)^2} = \frac{1}{4}$$

Repeat for each coordinate

$$y - 0 = \frac{1}{4}(x - 4)$$

$$y + 2 = -\frac{3}{8}(x - 4)$$

$$y - 2 = \frac{1}{8}(x - 4)$$

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