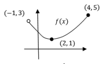
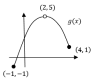
3.3 Extrema on an Interval

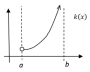
The maximum and minimum values (max and min) of a function are called the **extreme values** or **extrema** (singular: extremum) of a function. The process used to finding them is referred to as **optimization**. There will be times when we want to find the max and min for x on the entire domain, and at other times on a particular interval.



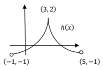
Abs. max. (4,5)
Abs. min. (2,1)



Abs. max. none
Abs. min. (-1,-1)



Abs. max. none
Abs. min. none



Abs. max. (3,2)
Abs. min. none

Defining Relative and Absolute Extrema of a Function

Relative Extrema – Points on the graph of a function where the function changes from increasing to decreasing & vice versa

Absolute Extrema – The highest & lowest points on the graph of a function or on a specified domain of a function.

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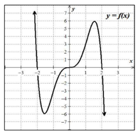
EX #1: In many cases we will be asked to analyze both open interval and closed interval functions. Consider the graphs of f and g shown below.

1A. Graph of $f(x)$



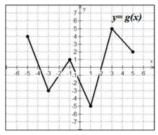
State the coordinates of the relative extrema of $f(x)$.

rel min ≈ (-1.5, -6)
rel max ≈ (1.5, 6)

Consider the domain of f, what are the absolute extrema of the function?

No absolute extrema Since function is unbounded.

1B. Graph of $g(x)$



State the coordinates of the relative extrema of $g(x)$.

rel. min (-3,-3) & (1,-5)
rel. max (-1,1) & (3,5)

Consider the domain of g, what are the absolute extrema of the function?

abs. max (3,5)
abs. min (1,-5)

1C. Considering both functions and their domains, where do the extrema occur, on the interval or at endpoints? Did the relative extrema occur at the function's absolute extrema?

Extrema can occur at endpoints or on the intervals. Absolute extremes can be relative extremes. on g(x) rel. extrema in a "neighborhood" was also an abs. extrema.

Let's investigate these two functions in a different way by changing the intervals of interest and limiting the domain of each function.

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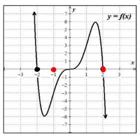
1D. Graph of $f(x)$



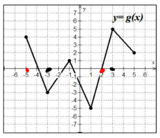
Find are the absolute extrema if we restrict the domain on the interval $-2 \le x \le 2$?

abs max (1.5, 6)
abs min (-1.5, -6)

Now, consider $-1 \le x \le 2$, what are the absolute extrema of the function?

abs. max (1.5, 6)
abs min (-1, -3)

1E. Graph of $g(x)$



What are the absolute extrema if we restrict the domain on the interval $-3 \le x \le 3$?

abs min (1,-5)
abs max (3,5)

Now, consider $-5 \le x \le 2$, what are the absolute extrema of the function?

abs min (1,-5)
abs max (-5,4)

1F. Are you paying attention? There are three distinct places at which absolute extrema can occur, given a restricted domain on a closed interval. Can you name the characteristics that are necessary? What features do you notice about the function?

Absolute Extrema can occur...
1) At the endpoints of an interval
2) where $f'(x)=0$ or
3) where $f'(x)$ does not exist

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CRITICAL NUMBER

is an x-value of the function x=c, in the domain a≤x≤b, where f'(c)=0 or where f'(c) dne (undefined)

EXTREME VALUE THEOREM

If a function f is continuous on a closed interval [a,b], then f(x) has at least one abs. max and at least one abs. min on the closed interval.

The only candidates for absolute extrema (max or min) must occur at a critical number or at an endpoint.

EX #2: For each function shown below, justify (explain) why the Extreme Value Theorem (EVT) can be applied, or why it does not apply on the given interval.

$f(x) = \frac{2x-3}{x-2}$ Interval: $-4 \le x \le 4$	EVT does not apply for f(x) on -4≤x≤4 b/c f(x) is not continuous at x=2.
$g(x) = e^{x-2} + 1$ Interval: $-5 \le x \le 5$	EVT applies for g(x) on -5≤x≤5 b/c g(x) is continuous for all reals.
$h(x) = x\sqrt{x+2}$ Interval: $-6 \le x \le 6$	EVT does not apply for h(x) on -6≤x≤6 b/c h(x) is continuous only x≥-2.

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Now we need to put these ideas together with a little direct practice. Let's develop an algebraic approach to what will be called ***THE CANDIDATES TEST*** for finding extrema.

EX #3: Find the absolute extrema of the function on the given interval, provided the EVT is applicable. If it is not, justify why.

A. $f(x) = 8x^3 - 3x^2 - 9x + 2$ on $[-1, 2]$

$f'(x) = 24x^2 - 6x - 9$
$(4x-3)(2x+1) = 0$
$x = 3/4 \quad x = -1/2$
$f(-1) = 0 \quad f(3/4) = -3.063$ min
$f(-1/2) = 4.75 \quad f(2) = 36$ max

B. $g(x) = x - 2\sin x$ on $[0, 2\pi]$

$g'(x) = 1 - 2\cos x$
$1 - 2\cos x = 0$
$-2\cos x = -1$
$\cos x = \frac{1}{2}$
$x = \pi/3, \ 5\pi/3$
$x=0 \quad x=\pi/3$
$g(0) = 0 \quad g(\pi/3) = -.685$ min
$g(5\pi/3) = 6.968$ max
$g(2\pi) = 6.28$

C. calculator active:
$h(x) = \sin^2 x - \cos x \qquad 0 \le x \le \frac{3\pi}{2}$

D. $k(x) = 4x^3 + 5x^2 - 42x + 7 \qquad 0 \le x \le 4$

[0, 4]
$k'(x) = 12x^2 + 10x - 42$
$0 = 2(6x^2 + 5x - 21)$
$(3x+7)(2x-3) = 0$
$\cancel{x = -7/3} \quad x = 3/2$
$k(0) = 7$
$k(3/2) = -31.25$ min
$k(4) = 175$ max

E. $f(x) = ln\,(x^2 - 9)$ on $[-2, 5]$

F. $h(x) = \sqrt[3]{(x-2)^2}$ on $[1, 10]$

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