RELATED RATES

The derivative, $\frac{dy}{dx}$, of a function, $y = f(x)$, is its instantaneous rate of change with respect to the variable x.

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- When a function describes either position or distance, its rate of change is interpreted as velocity.
- In general, a time rate of change answers the question: *How fast is a quantity changing?*
  - For example, if V is volume that is changing in time then dV/dt is the rate, or how fast, the volume is changing with respect to time.
  - If a person is walking toward a street lamp at a constant rate of 3 feet per second, then we know that the distance is decreasing, so $dx/dt = -3$ *ft/sec*.
  - If they walk away from the lamp the distance is increasing and the rate of change becomes positive $dx/dt = 3$ *ft/sec*.

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GUIDELINES FOR SOLVING RELATED RATE PROBLEMS:

1. Make a sketch and label the quantities.
2. Read the problem and identify all quantities as "KNOW," "GIVEN," and "FIND" with the appropriate information.
3. Write an equation involving the variables whose rates of change either are given or are to be determined.
4. Using the Chain Rule, implicitly differentiate both sides of the equation ***with respect to time, t.***
5. ***After completing Step 4***, substitute into the resulting equation all known values for the variables and their rates of change. Then, solve for the required rate of change.

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EX #1: Suppose x and y are both differentiable functions of t and are related by the equation $y = x^2 - 3x$. Find dy/dt when $x = 3$, given that $dx/dt = 2$ when $x = 3$.

$$dy/dt = 2x\frac{dx}{dt} - 3\frac{dx}{dt}$$
$$= 2(3)(2) - 3(2)$$
$$= 12 - 6$$
$$= 6$$

$$\left.dy/dt\right|_{x=3} = 6$$

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EX. #2:
A pebble is dropped into a calm pond, causing ripples in the form of concentric circles. The radius r of the outer ripple is increasing at a constant rate of 1 foot per second. When the radius is 4 feet, at what rate is the total area A of the disturbed water changing?

1. KNOW: $A = \pi r^2$

2. GIVEN: $dr/dt = 1$ ft/sec.
$r = 4$ ft

3. FIND: dA/dt

$$dA/dt = 2\pi r\, dr/dt$$
$$= 2\pi(4)(1)$$
$$= 8\pi \ \frac{ft^2}{sec}$$

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EX. #3:
Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet.

1. KNOW: $V = \frac{4}{3}\pi r^3$

2: GIVEN: $dV/dt = 4.5 \ ft^3/min$
$r = 2$ ft

3: FIND: dr/dt

$$dV/dt = 4\pi r^2 dr/dt$$
$$4.5 = 4\pi(2)^2 dr/dt$$
$$\frac{4.5}{16\pi} = \frac{\cancel{16\pi}\, dr/dt}{\cancel{16\pi}}$$
$$\frac{4.5}{16\pi} \ ft/min$$

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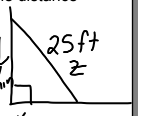
EX. #4:
The top of a 25-foot ladder is sliding down a vertical wall at a constant rate of 3 feet per minute. When the top of the ladder is 7 feet from the ground, what is the rate of change of the distance between the bottom of the ladder and the wall?



$y = 7ft$
$dy/dt = -3\,ft/min$

1. KNOW: $x^2 + y^2 = z^2$
2. GIVEN: $y = 7\,ft$ $z = 25ft$ $dy/dt = -3\,ft/min$ $dz/dt = 0$
3. FIND: $x,\ dx/dt$

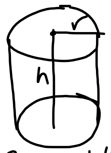
$7^2 + x^2 = 25^2$
$49 + x^2 = 625$
$\sqrt{x^2} = \sqrt{576}$
$x = 24$

$$\frac{2x\frac{dx}{dt} + 2y\,dy/dt = 2z\frac{dz}{dt}}{2}$$

$x\,dx/dt + y\,dy/dt = z\,dz/dt$
$24\,dx/dt + 7(-3) = 25(0)$
$\frac{\cancel{24}\,dx/dt}{\cancel{24}} = \frac{21}{24}$
$dx/dt = 7/8\ ft/min$

EX #6:
A right circular cylinder is changing shape. The radius is decreasing at a rate of 2 inches/sec while its height is increasing at the rate of 5 inches/sec. When the radius is 4 inches and the height is 6 inches, how fast is the volume changing?

Given: $V = \pi r^2 h$



1. KNOW: $V = \pi r^2 h$
2. GIVEN: $dr/dt = -2\,in/sec$ $dh/dt = 5\,in/sec$
$r = 4$
$h = 6$
3. FIND: dV/dt

$dV/dt = \pi r^2\,dh/dt + h(2\pi r)\,dr/dt$
$dV/dt = \pi(4)^2(5) + 6(2\pi(4))(-2)$
$= 80\pi - 96\pi$
$dV/dt = -16\pi\ in^3/sec.$