

Name _____
Optimization Worksheet #2

1. An open cylinder (has a bottom, but no top) has a volume of π cubic feet. What dimensions minimize the surface area?

$$\begin{aligned} SA &= 2\pi r h + \pi r^2 \\ SA &= 2\pi r \left(\frac{\pi}{r}\right) + \pi r^2 \\ &= 2\pi^2 r + \pi r^2 \\ &= 16\pi r^{-1} + \pi r^2 \\ O &= -16\pi r^{-2} + 2\pi r \\ \frac{16\pi}{r^2} &= 2\pi r \\ \frac{16\pi}{r^2} &= \frac{2\pi r^3}{r} \Rightarrow \sqrt[3]{8} = r \\ r &= 2 \text{ ft} \quad h = \frac{8}{r^2} = 2 \text{ ft} \end{aligned}$$

2. Find the maximum volume of a cone with a slant height of 10 inches.

$$\begin{aligned} V &= \frac{1}{3} \pi r^2 h \\ V &= \frac{1}{3} \pi (100-h^2) h \\ V &= \frac{1}{3} \pi h (100-h^2) \\ V &= \frac{100\pi}{3} h - \frac{\pi}{3} h^3 \\ V &= \frac{100\pi}{3} h - \pi h^2 \end{aligned}$$

$$\begin{aligned} h^2 + r^2 &= 100 \\ r^2 &= 100 - h^2 \\ r &= \sqrt{100 - h^2} \\ r &= \sqrt{100 - \left(\frac{100}{3}\right)^2} \\ r &= \sqrt{100 - \frac{10000}{9}} \\ r &= \frac{10\sqrt{2}}{3} \end{aligned}$$

3. A 384 square meter plot of land is to be enclosed by a fence. If it is divided into two equal parts by a fence parallel to one pair of sides. What dimensions of the outer rectangle will minimize the amount of fence used?

$$\begin{aligned} V &= \frac{1}{3} \pi \left(100 - \left(\frac{100}{3}\right)^2\right) \left(\frac{100}{3}\right) \\ F &= 4x + 3y \quad \text{Primary} \\ \frac{384}{2y} &= \frac{2x}{y} \quad \text{Secondary} \\ F &= 4\left(\frac{192}{y}\right) + 3y \quad x = \frac{192}{y} \\ F &= 768y^{-1} + 3y \\ F &= -768y^2 + 3 \\ \frac{768}{y^2} &= \frac{3}{1} \quad x = \frac{192}{16} = 12 \text{ m} \\ y &= 16 \text{ m} \\ \frac{384}{2y} &= \frac{768}{3} \\ \frac{384}{y} &= \frac{768}{3} \\ y &= \sqrt{256} \end{aligned}$$

4. A closed rectangular box with a square base has a volume of 252 cubic feet. The wood for the bottom costs \$5.00 per square foot and the top costs \$2.00 per square foot. The sides cost \$3.00 per square foot. What dimensions will minimize the cost of the wood?

$$\begin{aligned} V &= x \cdot x \cdot y \\ 252 &= x^2 y \\ y &= \frac{252}{x^2} \\ 14x - 3,024x^{-2} &= 0 \\ \frac{3,024}{x^2} &= \frac{14x}{1} \\ \frac{14x^3}{x} &= 3,024 \rightarrow \sqrt[3]{216} = x \quad y = \frac{252}{36} = 7 \text{ ft} \\ x &= 6 \text{ ft} \quad 6 \text{ ft} \times 6 \text{ ft} \times 7 \text{ ft} \end{aligned}$$