The Fundamental Theorem of Calculus

We have now seen the two major branches of calculus:
1) differential (tangent line problem)
2) integral (area problem)

Leibniz and Newton independently discovered a connection between the two branches, stated informally, differentiation and (definite) integration are inverse operations.

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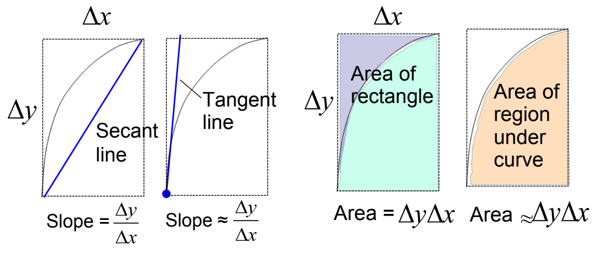

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THE FUNDAMENTAL THEOREM OF CALCULUS - PART I

Given that *f* represents the **RATE** at which the function *F* is changing, that is, $f(x) = F'(x)$ then

$\int_a^b f(x)dx$ represents the **TOTAL (NET) CHANGE IN *F* on the interval [*a, b*].**

If a function *f* is continuous on a closed interval [*a*, *b*] and *F* is an antiderivative of *f* on the interval [*a*, *b*], then

$$\int_a^b f(x)\,dx = F(b) - F(a)$$

If you like, you can also write this as: $\int_a^b g'(t)\,dt = g(b) - g(a)$

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In words we can say:

The **definite integral** of a function over [*a, b*] is equal to the **total change**, or **displacement**, in the antiderivative of *f* over the same interval. This is commonly known as the Fundamental Theorem of Calculus – Part I or the Evaluation Theorem.

When applying the Fundamental Theorem of Calculus, we will follow the notation below:

$$\int_a^b f(x)dx = F(x)\Big|_a^b$$
$$= F(b) - F(a)$$

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EX #1: TOTAL CHANGE CONCEPTS and explaining an integral in "real-world" terms.

A. If *f*(*t*) represents the rate at which your 401-K changes in value over the month of January.

Units of $f(t)$: $/day
Units of $d(t)$: days
Units of $f(t)\,dt$: $

$\int_1^{31} f(t)\,dt$ represents the total change in
The value of your 401-K in dollars for the month of January

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B. If $r(t)$ represents the rate at which your pizza changes temperature in a $475°F$ pizza oven 5 minutes after the pizza was had been placed into the oven to cook.

$\int_5^{20} r(t)\,dt$ represents the total change in
The temperature of the pizza in °F from $t=5$ to $t=20$.

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C. You and your friends are planning to buy a ticket to your favorite music group, *The New Tons*.

Everyone wants to hear their latest hit, "Why Integrate?". If $A(t)$ represents the rate at which students are arriving at the ticket window, and $P(t)$ represents the rate at which they buy their tickets and enter the concert, then:

1. $\int_1^5 A(t)\,dt$ represents the # of students arriving to get in line from $1 \le t \le 5$ (min)
2. $\int_1^5 P(t)\,dt$ represents the # of students who have bought tickets and leave the line on $1 \le t \le 5$

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3. $A(t)-P(t)$ is The rate at which the line is growing or shrinking
4. $\int_1^5 A(t)-P(t)\,dt$ represents the **total change in** The length of the line in students on the interval $1 \le t \le 5$ min

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D. Recall that velocity, $v(t)$ represents your velocity where the positive direction is to the right.

1. **1.** Velocity is a rate at which Position is changing.
2. **2.** $\int_5^{50} v(t)dt$ represents the **total change** in Position on [5, 50] which has a special name in math called displacement.
3. **3.** Likewise, $a(t)$ is the rate at which Velocity is changing.

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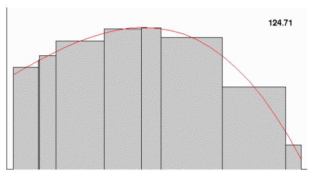
Riemann Integral Definition - Revisited



Notice that we are describing the area of a rectangle, with the width times the height, and we are adding the areas together. Each rectangle, by virtue of the Mean Value Theorem, describes an approximation of the curve section it is drawn over. Also notice that Δx_i need not be the same for all values of i, or in other words that the width of the rectangles can differ. What we have to do is approximate the curve with n rectangles. Now, as the size of the partitions get smaller and n increases, resulting in more partitions to cover the space, we will get closer and closer to the actual area of the curve.

By taking the limit of the expression as the norm of the partitions approaches zero, we arrive at the Riemann integral. We know that this limit exists because f was assumed to be integrable. That is, we take the limit as the largest of the partitions approaches zero in size, so that all other partitions are smaller and the number of partitions approaches infinity.

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EX #2: Find the value of the definite integrals by using the Evaluation Theorem stated above.

A. $\int_1^3 x^3 dx$

$\frac{x^4}{4}+C \Big|_1^3$

$\frac{3^4}{4}+C-\left(\frac{1^4}{4}+C\right)$

$=20$

$\frac{1}{4}(3^4-1^4)=20$

B. $\int_1^2 (x^2-3)dx$

$\frac{x^3}{3}-3x \Big|_1^2$

$\left(\frac{2^3}{3}-3(2)\right)-\left(\frac{1^3}{3}-3(1)\right)$

$-\frac{2}{3}$

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C. $\int_1^4 3\sqrt{x}\,dx$

$3\int_1^4 x^{\frac12}dx$

$3\left[\frac{x^{3/2}}{\frac32}\right]\Big|_1^4$

$3\left(\frac23 x^{3/2}\right)\Big|_1^4$

$2x^{3/2}\Big|_1^4$

$2(4)^{3/2}-2(1)^{3/2}$

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D. $\int_0^{\pi/4} sec^2 x\,dx$

$\tan x \Big|_0^{\pi/4}$

$\tan \pi/4 - \tan 0$

$1 - 0 = 0$

(1,0) $\sin 0 = 0$, $\cos 0 = 1$

	$\pi/6$	$\pi/4$	$\pi/3$
	30	45	60
sin	$\frac12$	$\frac{\sqrt2}{2}$	$\frac{\sqrt3}{2}$
cos	$\sqrt3/2$	$\sqrt2/2$	$\frac12$
tan	$\sqrt3/3$	1	$\sqrt3$

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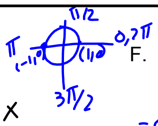
E. $\int_{-1}^{1}(2x-1)^2\,dx$



F. $\int_{0}^{\pi}(\sin x+1)\,dx$

$\int_{-1}^{1}(2x-1)(2x-1)\,dx$

$4x^2-4x+1 \rightarrow$

$\left.\frac{4x^3}{3}-\frac{4x^2}{2}+x\right|_{-1}^{1}$

$\left(\frac{4}{3}(1)^3-2(1)^2+1\right)-\left(\frac{4}{3}(-1)^3-2(-1)^2-1\right)$

$\frac{4}{3}-2+1-\left(-\frac{4}{3}-2-1\right)$

$\frac{14}{3}$

$\left.-\cos x+x\right|_{0}^{\pi}$

$(-\cos\pi+\pi)-(-\cos 0+0)$

$(-(-1)+\pi-(-1+0))$

$1+\pi+1 \rightarrow \pi+2$

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G. $\int_{1}^{8}\sqrt{\frac{2}{x}}\,dx$

H. $\int_{1}^{4}\frac{x-2}{\sqrt{x}}\,dx$

$\int_{1}^{8}\frac{\sqrt{2}}{\sqrt{x}}\,dx$

$\int_{-1}^{8}\sqrt{2}\,x^{-\frac{1}{2}}\,dx$

$\left.\sqrt{2}\,\frac{x^{\frac{1}{2}}}{\frac{1}{2}}\right|_{1}^{8}$

$\left.2\sqrt{2}\,x^{\frac{1}{2}}\right|_{1}^{8}$

$\left(2\sqrt{2}(8)^{\frac{1}{2}}\right)-\left(2\sqrt{2}(1)^{\frac{1}{2}}\right)$

$2\sqrt{2}\cdot\sqrt{8}-2\sqrt{2}$

$2\sqrt{2}\cdot2\sqrt{2}-2\sqrt{2}$

$4\cdot2-2\sqrt{2}$

$8-2\sqrt{2}$

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