9/4/19

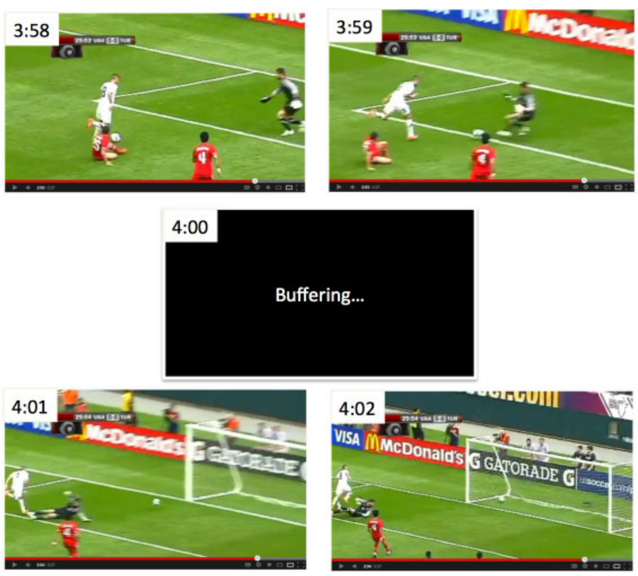
Understanding the Limit Graphically & Numerically

What is a limit?

Aug 28-10:08 PM

Key Analogy: Predicting A Soccer Ball

Pretend you're watching a soccer game. Unfortunately, the connection is choppy:



We missed what happened at 4:00. Even so, what's your prediction for the ball's position?

Easy. Just grab the neighboring instants (3:59 and 4:01) and predict the ball to be somewhere in-between.

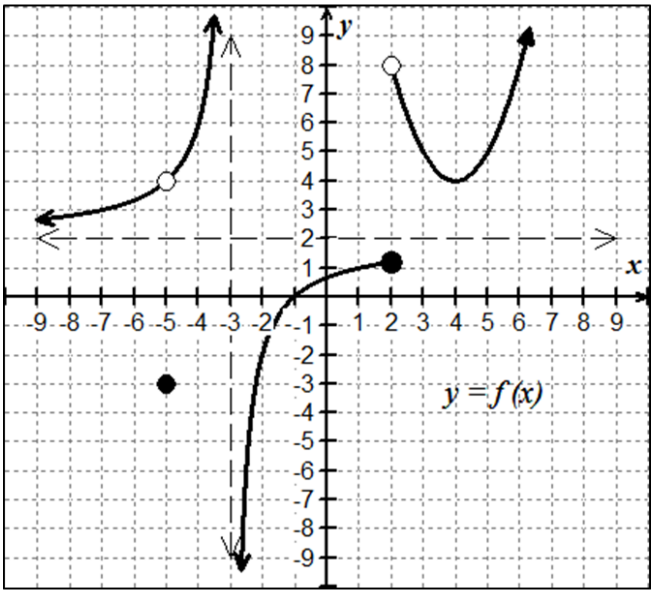
And… it works! Real-world objects don't teleport; they move through intermediate positions along their path from A to B. Our prediction is "At 4:00, the ball was between its position at 3:59 and 4:01". Not bad.

With a slow-motion camera, we might even say "At 4:00, the ball was between its positions at 3:59.999 and 4:00.001".

Sep 4-8:44 AM

Understanding the Limit Graphically and Numerically

Consider the graph of the function $f(x)$, graphed below:



Aug 8-7:31 PM

Using the graph, find the value of each of the following limits. If a limit does not exist, explain why.

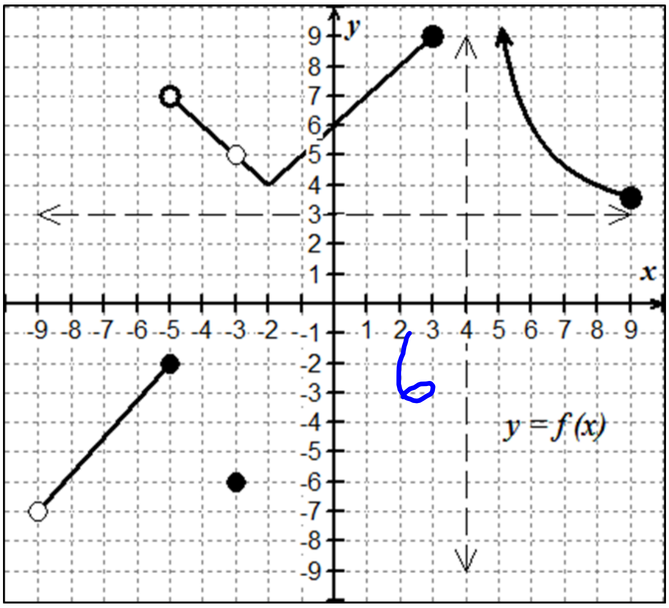
A. $\lim_{x\to -3^-} f(x)$ ∞	B. $\lim_{x\to -5} f(x)$ 4	C. $\lim_{x\to 4} f(x)$ 4
D.) $\lim_{x\to 2^+} f(x)$ 8	E.) $\lim_{x\to 2^-} f(x)$ 1	F.) $\lim_{x\to 2} f(x)$ dne differs on left & Rt
G.) $\lim_{x\to -1} f(x)$ 0	H.) $\lim_{x\to -\infty} f(x)$ 2	I.) $\lim_{x\to \infty} f(x)$ ∞

$\lim_{x\to -5}$ vs. $f(-5)$

4   -3

Aug 8-7:35 PM

Now you give it a try. Consider the graph shown below to find the value of each of the following limits. If a limit does not exist, explain why.



Aug 8-7:38 PM

A. $\lim_{x \to -5^+} f(x)$ 7	B. $\lim_{x \to -2} f(x)$ 4	C. $\lim_{x \to -3} f(x)$ 5
D.) $\lim_{x \to 3^+} f(x)$ dne no function as $x \to 3^+$	E.) $\lim_{x \to 3^-} f(x)$ 9	F.) $\lim_{x \to -5^-} f(x)$ -2
G.) $\lim_{x \to 0} f(x)$ 6	H.) $\lim_{x \to -9} f(x)$ dne no function as $x \to -9^-$	I.) $\lim_{x \to 4^+} f(x)$ ∞

Aug 8-7:40 PM

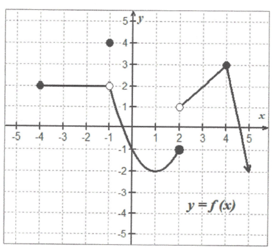
Limits are the "backbone" of understanding that connects algebra and geometry to the mathematics of calculus. In basic terms, a limit is just a statement that tells you what height a function *INTENDS TO REACH* as you get close to a specific *x*-value. Recall from Pre-Calculus that you evaluated three types of limits. Complete the table below:

PROPER LIMIT NOTATIONS		
TYPE OF LIMIT	PROPER NOTATION	VERBALLY:
Right-hand limit	$\lim_{x \to c^+} f(x)$	limit as x approaches c from the right
Left-hand limit	$\lim_{x \to c^-} f(x)$	limit as x approaches c from the left
General limit	$\lim_{x \to c} f(x)$	limit as x approaches c

Jul 2-8:13 AM

HW Solutions

You will use this graph to explore the limits for the problems on the next page



1. $f(2)$ = -1	2. $f(-1)$ = 4
3. $\lim_{x \to 4^-} f(x)$ = 3	4. $\lim_{x \to 2^+} f(x)$ = 1
5. $\lim_{x \to 2^-} f(x)$ = -1	6. $\lim_{x \to -1^+} f(x)$ = 2
7. $\lim_{x \to -1^-} f(x)$ = 2	8. $\lim_{x \to -4^+} f(x)$ = 2
9. $\lim_{x \to -4^-} f(x)$ dne	10. $\lim_{x \to -1} f(x)$ = 2
11. $\lim_{x \to 2} f(x)$ dne	12. $\lim_{x \to 5} f(x)$ = -2
13. $\lim_{x \to 0} f(x)$ = -1	14. $\lim_{x \to 1} f(x)$ = -2

Sep 5-8:06 AM

Consider the function shown below.
Say you want to find $\lim_{x\to 4^+} f(x)$, the positive sign in the limit notation indicates a right-hand limit. If you think of the function as a highway and imagine you are traveling along the graph of $f(x)$ toward $x=4$ FROM THE RIGHT, NOT TO THE RIGHT, and you stop at the vertical line $x=4$, the *y*-value where you stop is 3. Therefore, $\lim_{x\to 4^+} f(x)=3$.

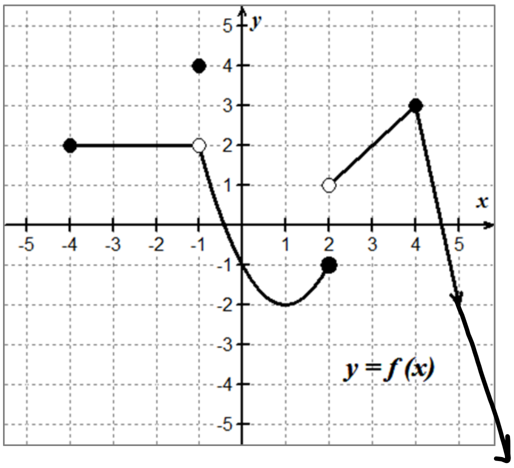


Figure 1-1

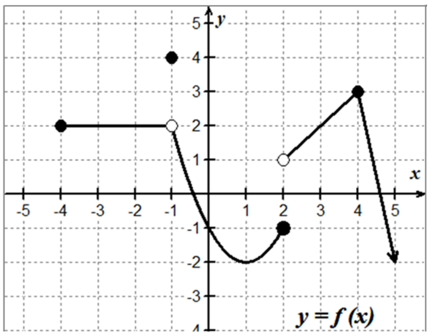
You will use this graph to explore the limits for the problems on the next page.

Jul 2-8:20 AM

EX #1: Use *Figure 1-1* to find the function values and evaluate each of the following limits:

1. $f(2)$	2. $f(1)$
3. $\lim_{x\to 4^-} f(x)$	4. $\lim_{x\to 2^+} f(x)$
5. $\lim_{x\to 2^-} f(x)$	6. $\lim_{x\to -1^+} f(x)$
7. $\lim_{x\to -1^-} f(x)$	8. $\lim_{x\to -4^+} f(x)$
9. $\lim_{x\to -4^-} f(x)$	10. $\lim_{x\to -1} f(x)$
11. $\lim_{x\to 2} f(x)$	12. $\lim_{x\to 5} f(x)$
13. $\lim_{x\to 0} f(x)$	14. $\lim_{x\to 1} f(x)$

Jul 2-8:27 AM



EX #2: THINK ABOUT THIS!

If we think of the function as a highway, then the point at $(2,-1)$ could be considered the end of the road, while the point at $(-1,2)$ is more like a "pothole." How would you describe the points located at

$(2,1)$ Dead End

$(4,3)$ Turn in the road

Hopefully, this analogy gives you a visual reference for understanding limits from a graphical approach. Let's get a little more formal with our definition now.

When finding limits, ask yourself, "What is happening to *y* as *x* gets close to a certain number?" You are finding the ***y*-value** for which the function is approaching as *x* approaches *c*.

Jul 2-8:30 AM

LIMIT EXISTENCE THEOREM:

$\lim_{x \to c} f(x)$ exists if and only if...

$$\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x) = L$$

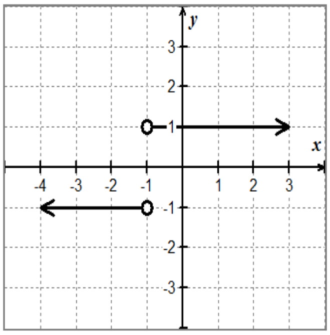
real #

Verbally: The limit as *x* approaches *c* on $f(x)$ will exist if and only if the limit as *x* approaches *c* from the left is equal to the limit as *x* approaches *c* from the right.

Jul 2-8:50 AM

EX #3: LIMITS CAN FAIL TO EXIST IN THREE SITUATIONS:

CASE 1: limits differ from left & right



Justify why the limit does not exist at $x = -1$ for $f(x) = \frac{|x+1|}{x+1}$

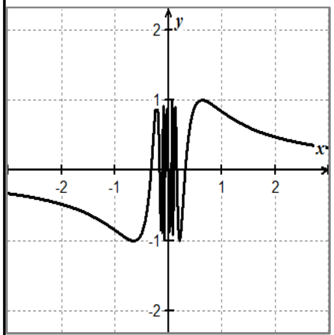
$\lim_{x \to -1^-} = -1$

$\lim_{x \to -1^+} = 1$

> DNE

not the same

CASE 2: Oscillating Behavior



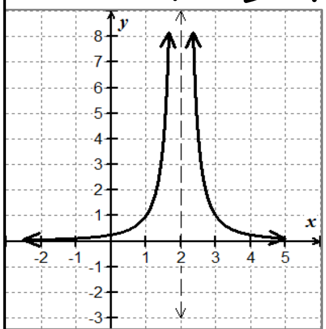
Justify why the limit does not exist at $x = 0$ for $f(x) = \sin\left(\frac{1}{x}\right)$

$\lim_{x \to 0^+} f(x) < 0$

$\lim_{x \to 0^-} f(x) > 0$

Jul 2-9:00 AM

CASE 3: limits of ∞ or $-\infty$



Justify why the limit does not exist at $x = 2$ for $f(x) = \frac{1}{(x-2)^2}$

$\lim_{x \to 2^-} f(x) = \infty$

$\lim_{x \to 2^+} f(x) = \infty$

> not a real #

EX #4: YOU OWN IT! In the box below, complete the sentence in your own words.

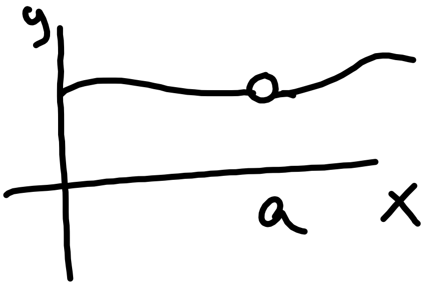
In order for the GENERAL LIMIT to exist, the function:

From left = From right

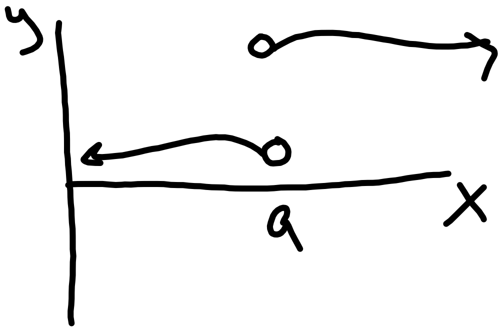
Jul 2-9:04 AM

EX #5: Sketch a graph to satisfy each set of conditions.

1. $f(a)$ is undefined (open circle)
2. $x=a$ is a point discontinuity
3. $\lim\limits_{x\to a} f(x)$ exists



1. $\lim\limits_{x\to a} f(x)$ DNE
2. $x=a$ is a jump discontinuity
3. $f(a)$ is undefined



Jul 2-9:10 AM

EX #6: Finding limits from a table of values

Now consider the function $f(x)=\dfrac{x-3}{x^2+2x-15}$.

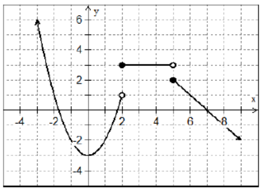
Complete the table below to find the limit as $x\to 3$.

x	2.9	2.99	2.999	3	3.001	3.01	3.1
$f(x)$							

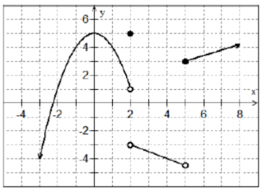
Based on your analysis, what are the values of each of the limits below?

$\lim\limits_{x\to 3^-} f(x) =$	$\lim\limits_{x\to 3^+} f(x) =$	$\lim\limits_{x\to 3} f(x) =$

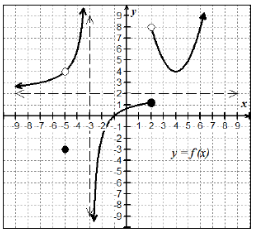
Jul 2-11:01 AM



A. $\lim_{x\to 0} f(x)$	B. $\lim_{x\to 2^-} f(x)$
C. $\lim_{x\to 4} f(x)$	D. $\lim_{x\to 5} f(x)$
E. $\lim_{x\to -\infty} f(x)$	F. $\lim_{x\to 7} f(x)$

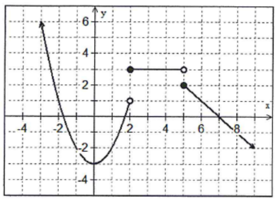


A. $\lim_{x\to 5^-} f(x)$	B. $\lim_{x\to 2} f(x)$
C. $\lim_{x\to 0} f(x)$	D. $\lim_{x\to \infty} f(x)$
E. $\lim_{x\to 4} f(x)$	F. $\lim_{x\to 2^+} f(x)$

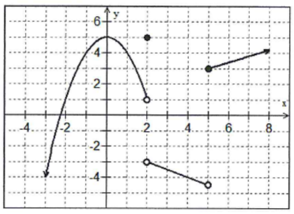


A. $\lim_{x\to -3^-} f(x)$	B. $\lim_{x\to -3^+} f(x)$
C. $\lim_{x\to 2^-} f(x)$	D.) $\lim_{x\to 2^+} f(x)$
E.) $\lim_{x\to \infty} f(x)$	F.) $\lim_{x\to -\infty} f(x)$
G.) $\lim_{x\to -3} f(x)$	H.) $\lim_{x\to 2} f(x)$

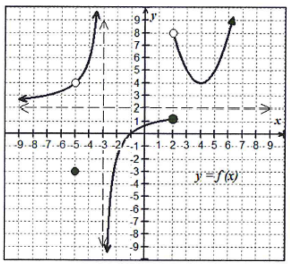
Sep 4-8:59 AM



A. $\lim_{x\to 0} f(x)$ -3	B. $\lim_{x\to 2^-} f(x)$ 1
C. $\lim_{x\to 4} f(x)$ 3	D. $\lim_{x\to 5} f(x)$ dne $\lim_{x\to 5^+} f(x) \neq \lim_{x\to 5^-} f(x)$
E. $\lim_{x\to -\infty} f(x)$ ∞	F. $\lim_{x\to 7} f(x)$ 0



A. $\lim_{x\to 5^-} f(x)$ -4.5	B. $\lim_{x\to 2} f(x)$ dne $\lim_{x\to 2^+} f(x) \neq \lim_{x\to 2^-} f(x)$
C. $\lim_{x\to 0} f(x)$ 5	D. $\lim_{x\to \infty} f(x)$ ∞
E. $\lim_{x\to 4} f(x)$ -4	F. $\lim_{x\to 2^+} f(x)$ -3



A. $\lim_{x\to -3^-} f(x)$ ∞	B. $\lim_{x\to -3^+} f(x)$ $-\infty$
C. $\lim_{x\to 2^-} f(x)$ 1	D.) $\lim_{x\to 2^+} f(x)$ 8
E.) $\lim_{x\to \infty} f(x)$ ∞	F.) $\lim_{x\to -\infty} f(x)$ 2
G.) $\lim_{x\to -3} f(x)$ dne $\lim_{x\to -3^-} f(x) \neq \lim_{x\to -3^+} f(x)$	H.) $\lim_{x\to 2} f(x)$ dne $\lim_{x\to 2^-} f(x) \neq \lim_{x\to 2^+} f(x)$

Sep 6-8:25 AM