Name _____ Calculus Q1 T1

Review of limits and continuity

1. Determine the value of c that makes the piecewise-defined function $g(x)$ everywhere continuous. $g(x)=\begin{cases}\sqrt{x+4}, & x<5\\ x^2+c, & x\ge 5\end{cases}$

let $x=5$

$\sqrt{x+4}=x^2+c$

$\sqrt{5+4}=5^2+c$

$3=25+c$

$\boxed{c=-22}$

2. Is $h(x)$ continuous for all real numbers? If so show why.

$h(x)=\begin{cases}x+3, & x\le -1\\ -x^2, & x>-1\end{cases}$

$\lim_{x\to -1^-} -1+3=2$ $f(-1)=2$

$\lim_{x\to -1^+} -(-1)^2=-1$

NO $\lim_{x\to -1}$ dne

3. Evaluate $\lim_{h\to 0}\frac{(2+h)^2-4}{h}$.

$\frac{(2+h)(2+h)-4}{h}\to\frac{4+4h+h^2-4}{h}\to\frac{\cancel{h}(4+h)}{\cancel{h}}\to 4+0\to\boxed{4}$

4. Evaluate $\lim_{x\to 4}\frac{x-4}{\sqrt{x^2-16}}$. $\cdot\frac{\sqrt{x^2-16}}{\sqrt{x^2-16}}\to\frac{\cancel{(x-4)}(\sqrt{x^2-16})}{\cancel{x^2-16}}\to\frac{0}{8}\to\boxed{0}$

$(\cancel{x-4})(x+4)$

5. Evaluate $\lim_{x\to 0}\frac{2^x-1}{2^x}$.

$\frac{2^0-1}{2^0}\to\frac{1-1}{1}\to\boxed{0}$

6. Evaluate $\lim_{x\to 2}\frac{x^2+3x-10}{x-2}$. $\frac{(x+5)\cancel{(x-2)}}{\cancel{(x-2)}}\to\boxed{7}$

7. Evaluate $\lim_{x\to\infty}\frac{2x^3-3}{3x^3+25}$. $\to\boxed{2/3}$

$\frac{\frac{2\cancel{x^3}}{\cancel{x^3}}-\frac{3}{x^3}}{\frac{3\cancel{x^3}}{\cancel{x^3}}+\frac{25}{x^3}}$

$\frac{2-0}{3+0}\to\frac{2}{3}$

8. Evaluate $\lim_{x\to\infty}\frac{2x^7+3x}{3x^5+2x}$. $\boxed{\infty}$

$\frac{\frac{2x^7}{x^5}+\frac{3x}{x^5}}{\frac{3x^5}{x^5}+\frac{2x}{x^5}}\to\frac{2x^2+0}{3+0}$

9. Evaluate $\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2}\cdot\frac{(\sqrt{x^2+3}+2)}{\sqrt{x^2+3}+2}$

$\frac{\cancel{(x-1)}(\sqrt{x^2+3}+2)}{x^2+3-4}\to\frac{4}{2}\to\boxed{2}$

$\cancel{x^2-1}\ \cancel{(x+1)(x-1)}$

10. Evaluate $\lim_{x\to 4}\frac{x+14}{\sqrt{x^2-7}}$.

$\frac{4+14}{\sqrt{16-7}}\to\frac{18}{3}\to\boxed{6}$

In # 11 – 20, which of the statements are true about the function y = f(x) graphed and which are false?

11. Find $\lim_{x \to -1^-} f(x) = 1$. T

12. Find $\lim_{x \to 2} f(x) = DNE$. F

13. Find $\lim_{x \to 2} f(x) = 2$. F

14. f(2) = 2 T

15. Find $\lim_{x \to 1^+} f(x) = 2$. T

16. Find $\lim_{x \to 1^-} f(x) = 1$. T

17. Find $\lim_{x \to 1} f(x) = DNE$. T

18. Find $\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$. T

19. $\lim_{x \to c} f(x)$ exists at every c in (-1,1) T

[note: () means that the endpoints are not included].

20. $\lim_{x \to c} f(x)$ exists at every c in (1,3) T

[note: () means that the endpoints are not included].