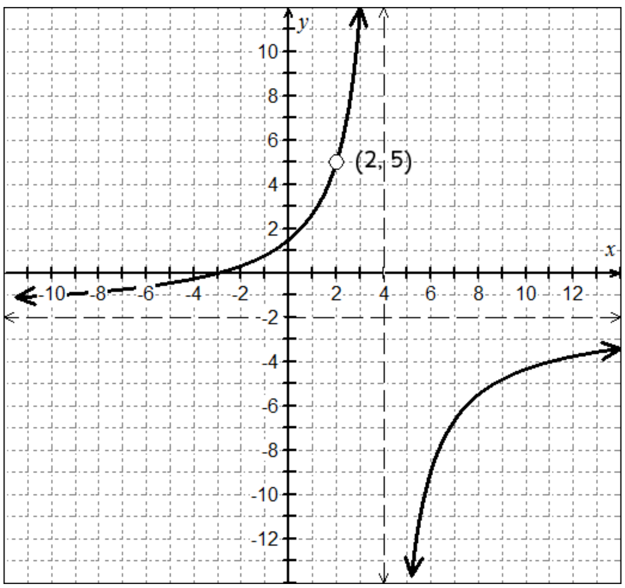
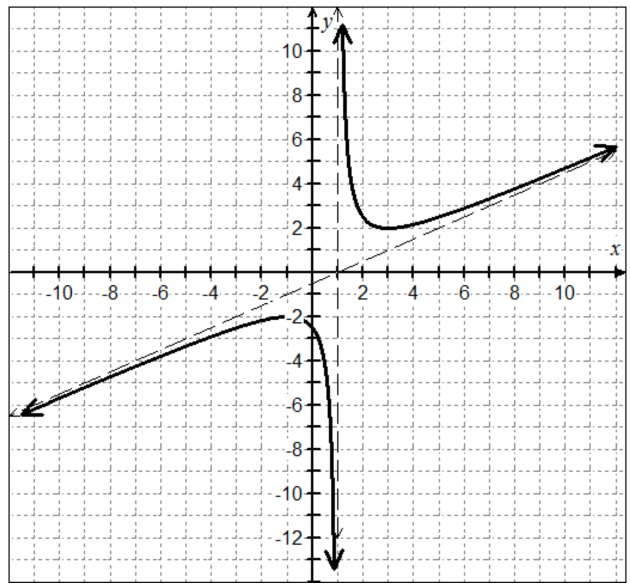
Infinite Limits and Limits at Infinity

In the lesson on <u>*Understanding Limits*</u> you were confronted with these two situations. Here we begin to **compare and contrast** the behavior of functions as they *approach infinity*, as well as, functions that *tend toward infinity* in certain circumstances. Let's go…

EX #1: Use the graphs of $f(x)$ and $g(x)$ shown below to compare and contrast behaviors involving infinity.

$$f(x) = \frac{-2x^2 - 2x + 12}{x^2 - 6x + 8} \qquad g(x) = \frac{x^2 - 2x + 5}{2x - 2}$$





An Infinite Limit is:	A Limit at Infinity is:
$\lim_{x \to 4^-} f(x) =$	$\lim_{x \to -\infty} f(x) =$
$\lim_{x \to 4^+} f(x) =$	$\lim_{x \to \infty} f(x) =$
$\lim_{x \to 1^-} g(x) =$	$\lim_{x \to -\infty} g(x) =$
$\lim_{x \to 1^+} g\,(x) =$	$\lim_{x \to \infty} g(x) =$
Discovery:	Discovery:

In Pre-Calculus you learned some basic truths about rational functions.

1. When a factor cancelled from the denominator a ______________________________ occurred.

2. When a factor would not cancel from the denominator a __________________________ occurred.

EX #2: Use the previous equations to find the limits analytically.

A. $\lim\limits_{x \to 4^-} \frac{-2x^2-2x+12}{x^2-6x+8}$	**B.** $\lim\limits_{x \to 4^+} \frac{-2x^2-2x+12}{x^2-6x+8}$
C. $\lim\limits_{x \to 1^-} \frac{x^2-2x+5}{2x-2}$	**D.** $\lim\limits_{x \to 1^+} \frac{x^2-2x+5}{2x-2}$

Definition and Justification of Vertical Asymptotes

CASE 1: $h(c) = \frac{non-zero}{zero}$ $x = c$ is:	**CASE 2:** $h(c) = \frac{zero}{zero}$ $x = c$ is:

IN CALCULUS, YOU MUST USE NEW LANGUAGE IN ORDER TO JUSTIFY!

LIMIT DEFINITION (JUSTIFICATION) OF A VERTICAL ASYMPTOTE

EX #3: Use the function below to find any vertical asymptote(s) that exist. Justify your answer using limits.

$$h(x) = \frac{2x^2 + 9x - 5}{x^2 + 3x - 10}$$

Limits at Infinity

Next, we will explore **limits at infinity** in order to differentiate between the two conditions. Recall the lessons from Pre-Calculus related to analyzing the **end behavior of functions.** In the exercise below, use this prior knowledge to find each limit at infinity.

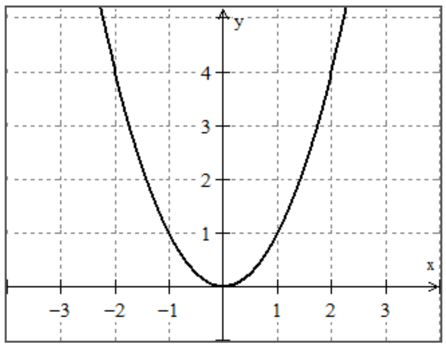
EX #4: Find each limit at infinity, explain your thinking.

A. $\lim_{x \to \infty} (x^2 - 4)(x^2 + 3)$	**B.** $\lim_{x \to -\infty} (5x^3 - 2x + 4)$
C. $\lim_{x \to \infty} \frac{3x^2 - 4}{x^2 + 1}$	**D.** $\lim_{x \to -\infty} \frac{5x - 2}{x^2 + 1}$

EX #5: There are only four possible outcomes when you explore behavior to the extreme right or left.

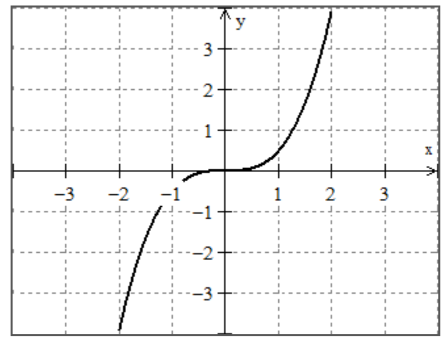
A. The curve can increase without bound.

$$\lim_{x \to \infty} f(x) =$$



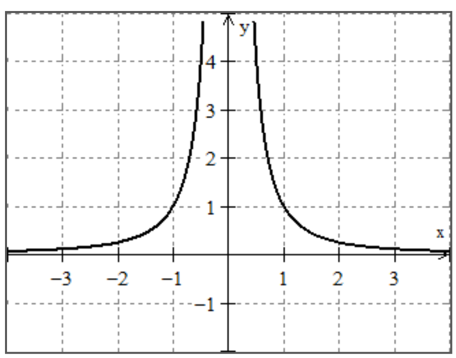
B. The curve can decrease without bound.

$$\lim_{x \to -\infty} f(x) =$$



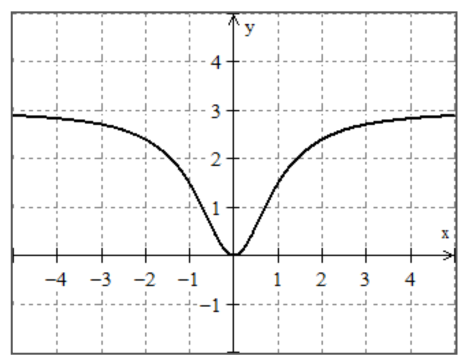
C. The curve can become asymptotic to the x-axis.

$$\lim_{x \to \infty} f(x) =$$



D. The curve can become asymptotic to a specific y-value.

$$\lim_{x \to -\infty} f(x) =$$



Revisiting the rules for finding potential horizontal asymptotes for rational functions from Pre-Calculus, you can use the idea of a limit and calculus to see why those rules hold true.

1. If degree of numerator is less than degree of denominator (bottom heavy), then limit is zero.

$$\lim_{x \to \pm\infty} f(x) = 0$$

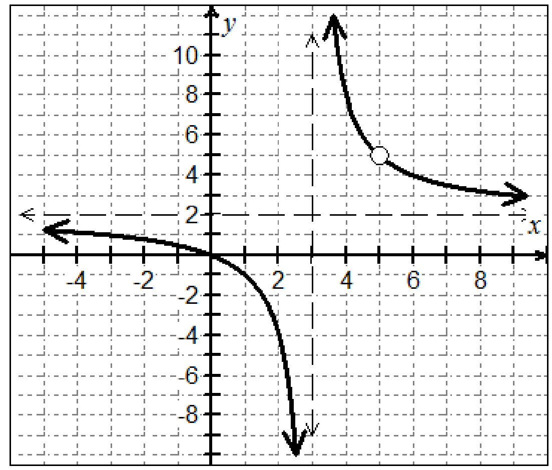
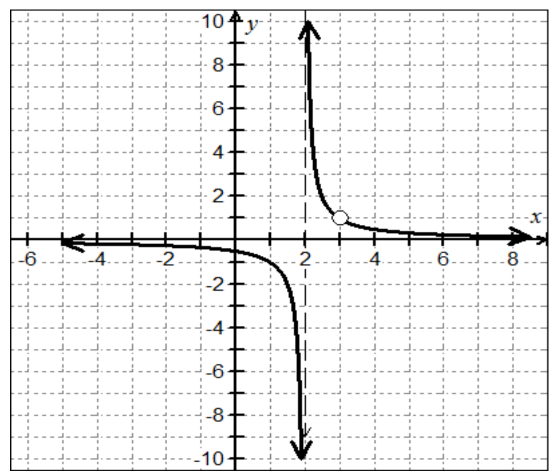
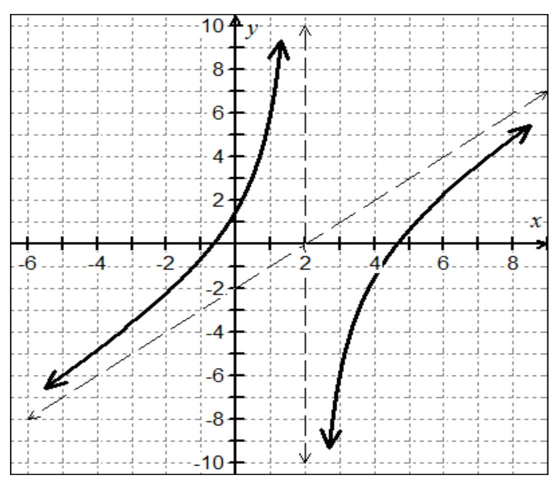
2. If degree of numerator equals degree of denominator (powers equal), then limit is the ratio of coefficients of the highest degree.

$$\lim_{x \to \pm\infty} f(x) = \frac{\text{coefficient of numerator's highest power}}{\text{coefficient of denominator's highest power}}$$

3. If degree of numerator is greater than degree of denominator (top heavy*), then limit does not exist.

$$\lim_{x \to \pm\infty} f(x) = \pm\infty$$

EX #6: Divide every term in the rational expression by the ***highest power of x that appears in the denominator.*** Then, apply the Properties of Limits to evaluate each "piece" to find the limit at infinity, end behavior.

A. $\lim\limits_{x \to \infty} \frac{2x^2 - 10x}{x^2 - 8x + 15}$	
B. $\lim\limits_{x \to \infty} \frac{x + 3}{x^2 + 4x + 5}$	
C. $\lim\limits_{x \to \infty} \frac{x^2 - 4x - 3}{x - 2}$	

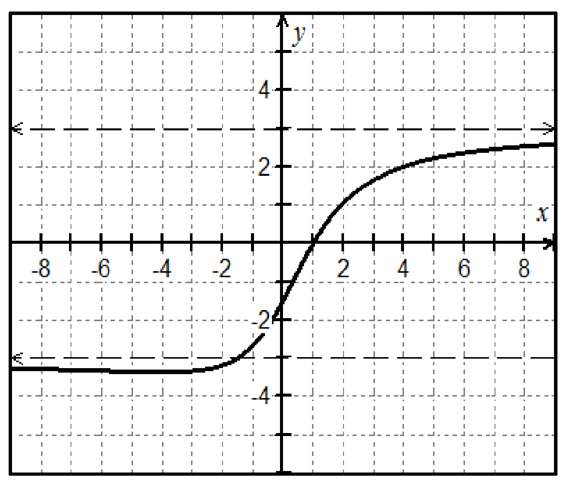
EX #7: Summarize and discuss characteristics and end behavior at horizontal asymptotes and slant asymptotes based on your observations in EX #6.

Functions with Horizontal Asymptotes	Functions with Slant Asymptotes

IN CALCULUS, YOU MUST USE NEW LANGUAGE IN ORDER TO JUSTIFY!

LIMIT DEFINITION (JUSTIFICATION) OF A HORIZONTAL ASYMPTOTE

EX #8: Use algebraic techniques to find the limits for $g(x) = \frac{3x-3}{\sqrt{x^2+4}}$, whose graph is shown.



$\lim\limits_{x \to -\infty} \frac{3x - 3}{\sqrt{x^2 + 4}}$	$\lim\limits_{x \to \infty} \frac{3x - 3}{\sqrt{x^2 + 4}}$