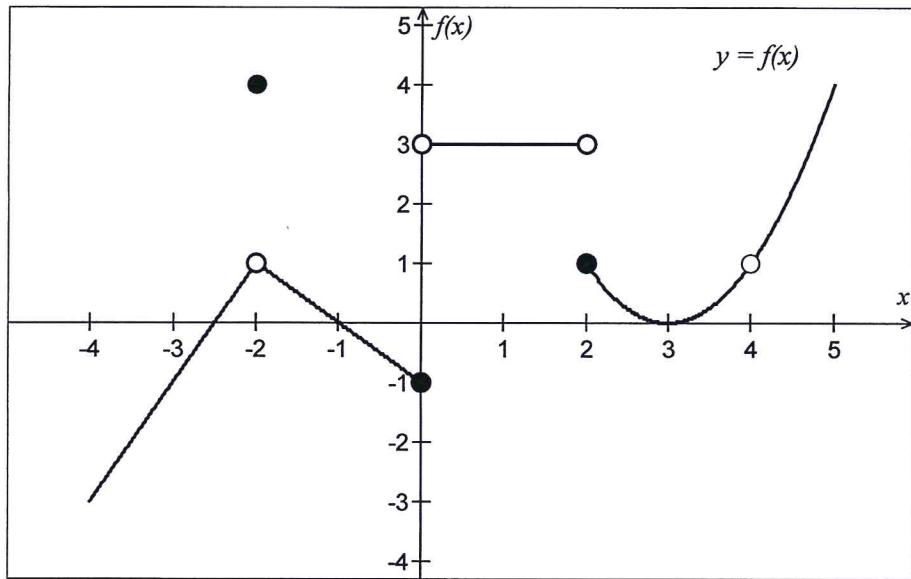


Problems 1–10, use the graph of the function $y = f(x)$ to estimate the limits.



1. A) $\lim_{x \rightarrow -1^-} f(x) = 0$ B) $\lim_{x \rightarrow -1^+} f(x) = 0$ C) $\lim_{x \rightarrow -1} f(x) = 0$
2. A) $\lim_{x \rightarrow 0^-} f(x) = -1$ B) $\lim_{x \rightarrow 0^+} f(x) = 3$ C) $\lim_{x \rightarrow 0} f(x) = \text{dne}$
3. A) $\lim_{x \rightarrow 2^-} f(x) = 3$ B) $\lim_{x \rightarrow 2^+} f(x) = 1$ C) $\lim_{x \rightarrow 2} f(x) = \text{dne}$
4. A) $\lim_{x \rightarrow -2^-} f(x) = 1$ B) $\lim_{x \rightarrow -2^+} f(x) = 1$ C) $\lim_{x \rightarrow -2} f(x) = 1$
5. A) $\lim_{x \rightarrow 4^-} f(x) = 1$ B) $\lim_{x \rightarrow 4^+} f(x) = 1$ C) $\lim_{x \rightarrow 4} f(x) = 1$
6. A) $\lim_{x \rightarrow -1} f(x) = 0$ B) $f(-1) = 0$ C) Is f continuous at $x = -1$?
Why?
Yes; $\lim_{x \rightarrow -1} f(x) = f(-1)$
7. A) $\lim_{x \rightarrow -2} f(x) = 1$ B) $f(-2) = 4$ C) Is f continuous at $x = -2$?
Why?
No; $\lim_{x \rightarrow -2} f(x) \neq f(-2)$
8. A) $\lim_{x \rightarrow 0} f(x) = \text{dne}$ B) $f(0) = -1$ C) Is f continuous at $x = 0$?
Why?
No; $\lim_{x \rightarrow 0} f(x) \text{ dne}$
9. A) $\lim_{x \rightarrow 2} f(x) = \text{dne}$ B) $f(2) = 1$ C) Is f continuous at $x = 2$?
Why?
No; $\lim_{x \rightarrow 2} f(x) \text{ dne}$
10. A) $\lim_{x \rightarrow 4} f(x) = 1$ B) $f(4) = \text{dne}$ C) Is f continuous at $x = 4$?
Why?
No; $\lim_{x \rightarrow 4} f(x) \neq f(4)$
Jean Adams

Problems 11 – 14, classify the discontinuities of each function below as removable, jump, or infinite.

11. $f(x) = \begin{cases} x^2 - 1 & x \geq 1 \\ 4 - x & x < 1 \end{cases}$ $\lim_{x \rightarrow 1^-} f(x) = 3 \quad \lim_{x \rightarrow 1^+} f(x) = 0$ jump discontinuity	12. $h(x) = \begin{cases} x - 3 & x \neq 2 \\ -4 & x = 2 \end{cases}$ $\lim_{x \rightarrow 2} h(x) = 1$ $h(2) = -4$ removable discontinuity
13. $g(x) = \begin{cases} 3 - x & x \geq 1 \\ x^3 & x < 1 \end{cases}$ $\lim_{x \rightarrow 1^-} g(x) = 1 \quad \lim_{x \rightarrow 1^+} g(x) = 2$ jump discontinuity	14. $f(x) = \begin{cases} x + 1 & x < 2 \\ -1 & x = 2 \\ x^2 + 1 & x > 2 \end{cases}$ $\lim_{x \rightarrow 2^-} f(x) = 3 \cdot \lim_{x \rightarrow 2^+} f(x) = 5$ $f(2) = -1$ jump discontinuity

Problems 15 – 16, use the three part definition of continuity to determine if the given functions are continuous at the indicated values of x .

15. $f(x) = \begin{cases} e^x \cos x, & x \geq \pi \\ e^x \tan\left(\frac{3x}{4}\right), & x < \pi \end{cases}$ at $x = \pi$ 1) $f(\pi) = e^\pi \cos \pi = -e^\pi$ 2) $\lim_{x \rightarrow \pi^-} e^x \tan\left(\frac{3x}{4}\right) = -e^\pi$ $\lim_{x \rightarrow \pi^+} e^x \cos x = -e^\pi$ $\lim_{x \rightarrow \pi} f(x)$ exists 3) $f(\pi) = \lim_{x \rightarrow \pi} f(x)$ $\therefore f(x)$ is continuous at $x = \pi$	16. $g(x) = \begin{cases} \frac{x^2 - 9}{x+3} & x \neq -3 \\ 5 & x = -3 \end{cases}$ at $x = -3$ 1) $g(-3) = 5$ 2) $\lim_{x \rightarrow -3^-} \left(\frac{x^2 - 9}{x+3}\right) = -6$ $\lim_{x \rightarrow -3^+} \left(\frac{x^2 - 9}{x+3}\right) = -6$ $\lim_{x \rightarrow -3} g(x)$ exists 3) $g(-3) \neq \lim_{x \rightarrow -3} g(x)$ $\therefore g(x)$ is not continuous at $x = -3$
--	---

17. Consider the function $f(x)$ to answer the questions. $f(x) = \begin{cases} -3 & x \leq -1 \\ mx + k & -1 < x < 4 \\ 3 & x \geq 4 \end{cases}$

A. What two limits must be equal in order for the function to be continuous at $x = -1$?

$$\lim_{x \rightarrow -1^-} f(x) = \lim_{x \rightarrow -1^+} f(x) \Rightarrow mx + k = -3$$

$$-m + k = -3 \quad (x = -1)$$

B. What two limits must be equal in order for the function to be continuous at $x = 4$?

$$\lim_{x \rightarrow 4^-} f(x) = \lim_{x \rightarrow 4^+} f(x) \Rightarrow mx + k = 3$$

$$4m + k = 3 \quad (x = 4)$$

C. Find the values of m and k so that the function is continuous everywhere.

$$\begin{aligned} 4m + k &= 3 \\ -m + k &= -3 \end{aligned} \Rightarrow \begin{aligned} 4m + k &= 3 \\ -4m + 4k &= -12 \end{aligned}$$

$$\begin{aligned} 5k &= -9 \\ k &= -\frac{9}{5} \end{aligned} \quad \begin{aligned} -m &= -\frac{15}{5} + \frac{9}{5} \\ m &= \frac{6}{5} \end{aligned}$$

$$k = -\frac{9}{5}$$

$$m = \frac{6}{5}$$

Problems 18 – 21, find all value(s) of a , b , c or k that make the function continuous everywhere.

18. $f(x) = \begin{cases} kx^2 & x \leq 3 \\ 4x - 11 & x > 3 \end{cases}$

$$\begin{aligned} 9k &= 4(3) - 11 \\ 9k &= 1 \\ \underline{\underline{k}} &= \frac{1}{9} \end{aligned}$$

$$\lim_{x \rightarrow 3^-} f(x) = \lim_{x \rightarrow 3^+} f(x)$$

iff $k = \frac{1}{9}$, then

$$\lim_{x \rightarrow 3} f(x) = 1$$

$x = 3$
Target

19. $g(x) = \begin{cases} cx^2 & x < 1 \\ 4 & x = 1 \\ -x^3 + kx & x > 1 \end{cases}$

$$\begin{aligned} c &= 4 \\ -1 + k &= 4 \\ \underline{\underline{k}} &= 5 \end{aligned}$$

$$\lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^+} g(x)$$

iff $c = 4$ and $k = 5$

then $\lim_{x \rightarrow 1} g(x) = 4$

20. $h(x) = \begin{cases} x^2 + ax + b & x < 0 \\ 6x + 5 & 0 \leq x \leq 1 \\ x > 1 \end{cases}$

$\pi = 0^2 + 0 \cdot a + b$ @ $x=0$
 $b = \pi$

$1 + a + \pi = 6(1) + 5$ @ $x=1$
 $a + \pi = 10$
 $a = 10 - \pi$

$\therefore \lim_{x \rightarrow 0} h(x) = \pi$ iff
 $\lim_{x \rightarrow 1} h(x) = 11$
 $a = 10 - \pi$ and $b = \pi$

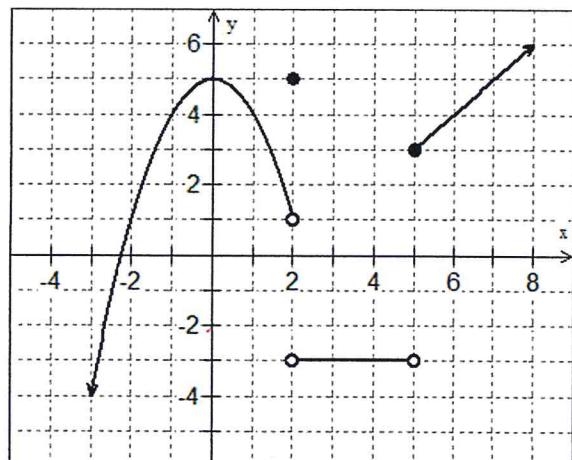
21. $f(x) = \begin{cases} x^2 & x < 1 \\ \sin(bx) & x \geq 1 \end{cases}$

$1 = \sin b$ @ $x=1$
 $\sin^{-1} 1 = b$
 $b = \pi/2$

$\therefore \lim_{x \rightarrow 1} f(x) = 1$
iff $b = \pi/2$

22. Write the piecewise function, $g(x)$, that defines the graph shown at right. Then, use the three part definition of continuity to analytically justify why $g(x)$ is discontinuous at $x = 2$ and $x = 5$.

$$g(x) = \begin{cases} 5-x^2 & x < 2 \\ 5 & x = 2 \\ -3 & 2 < x < 5 \\ x-2 & x \geq 5 \end{cases}$$



- 1) $g(2) = 5$
- 2) $\lim_{x \rightarrow 2^-} g(x) = 1$ but $\lim_{x \rightarrow 2^+} g(x) = -3$ so
 $\lim_{x \rightarrow 2} g(x)$ dne
- 3) $g(2) \neq \lim_{x \rightarrow 2} g(x)$ $\therefore g(x)$ is not continuous at $x=2$