

Name _____

Calculus

Review

This review sheet should NOT serve as your only review. You should review all notes and tests.

Questions 1 through 7 refer to the graph of $y = f(x)$ shown to the right.

1. $\lim_{x \rightarrow 1^-} f(x) = -\infty$

2. $\lim_{x \rightarrow 1^+} f(x) = \infty$

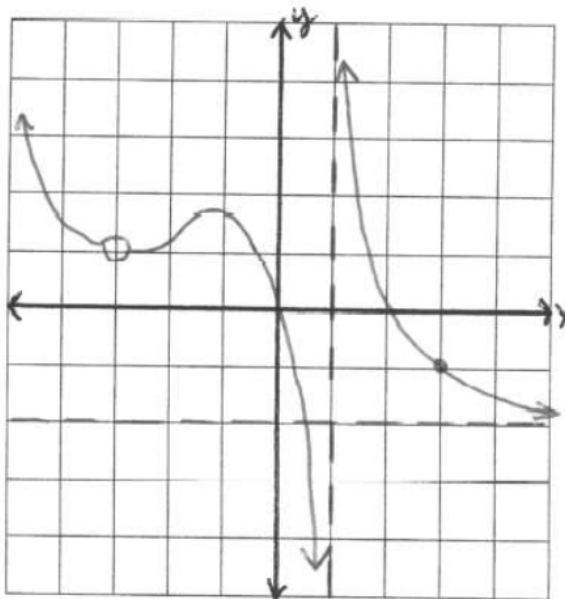
3. $\lim_{x \rightarrow 1} f(x) = \text{DNE}$

4. $\lim_{x \rightarrow -3} f(x) = 1$

5. $\lim_{x \rightarrow 3} f(x) = -1$

6. $\lim_{x \rightarrow -\infty} f(x) = \infty$

7. $\lim_{x \rightarrow \infty} f(x) = -2$



For each of the following functions, use the definition of derivative to find $f'(x)$.

$$\text{Recall: } f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

8. $f(x) = 2x^2 - 8x + 5$

9. $f(x) = \sqrt{3x+1}$

Find the derivative of each of the following:

10. $f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$

11. $f(x) = \sin^2(3x+1)$

12. $f(x) = \ln(\sin x)$

13. $f(x) = \ln(\sqrt{2x+3})$

14. $f(x) = \frac{e^{2x}}{x^2}$

15. $f(x) = \sqrt[4]{(x^2+5x)^3}$

16. $f(x) = \sqrt{x} \tan x$

17. $f(x) = x^3 \sec(e^{3x} - 1)$

18. $f(x) = e^{\sqrt{2x}}$

19. Find the slope of the line tangent to $y = x^2 \ln(3x)$ when $x = 1$.

$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{2(x+h)^2 - 8(x+h) + 5 - (2x^2 - 8x + 5)}{h}$$

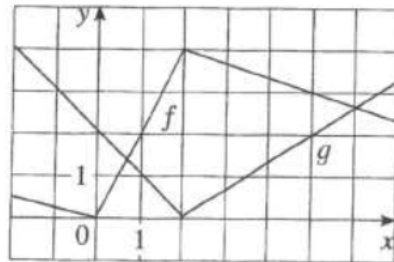
$$\lim_{h \rightarrow 0} \frac{2x^2 + 4xh + 2h^2 - \cancel{8x} - 8h + 5 - 2x^2 + \cancel{8x} - 5}{h}$$

$$\lim_{h \rightarrow 0} \frac{4xh + 2h^2 - 8h}{h} \rightarrow \frac{2h(2x+h-4)}{h}$$

$$2(2x+0-4) \rightarrow \boxed{4x-8}$$

20. Write the equation of the line tangent to $y = 3x^2 - 2x + 1$ when $x = -1$.
21. Write the equation of the normal to $y = 5 - x^2$ when $x = 2$.
22. An object moves along a line so that its position at time t is given by $s(t) = 2t^3 - 15t^2 + 24t - 10$ where $t \geq 0$.
- What is the object's position at time $t = 3$?
 - What is the object's velocity at time $t = 3$?
 - What is the object's acceleration at time $t = 3$?
 - Is the object speeding up or slowing down at $t = 3$? Justify your response.
 - When is the object at rest?
 - When is the object moving right?
 - How far does the object travel in the first 3 seconds?
23. If f and g are the functions shown below. Let $h(x) = f(g(x))$ and $s(x) = f(x)g(x)$.

Find: $h'(1)$ and $s'(1)$



24. The following table records the values of f , f' , g , and g' at $x = 1$, $x = 2$, and $x = 3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

If $n(x) = \frac{f(x)}{g(x)}$, $h(x) = f(g(x))$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

25. If $f(x) = \sqrt[3]{(x^2 - 2x - 1)^2}$, then $f'(0) =$

28. Find $\frac{dy}{dx}$ for the given curve: $x^3 + y^3 = 18y$

29. Find $\frac{dy}{dx}$ for the given curve: $x^2y - xy^2 = 4x$

30. Write the equation of the tangent to $x^2 - xy = y^2 + 1$ in the first quadrant when $y = 1$.

$$9. \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{3(x+h)+1} - \sqrt{3x+1})}{h} \cdot \frac{(\sqrt{3(x+h)+1} + \sqrt{3x+1})}{(\sqrt{3(x+h)+1} + \sqrt{3x+1})}$$

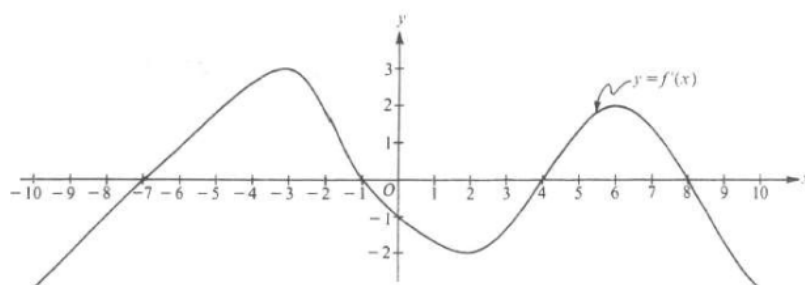
$$\frac{3(x+h)+1 - (3x+1)}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})} \rightarrow \frac{\cancel{3x} + \cancel{3h} + \cancel{1} - \cancel{3x} - \cancel{1}}{h(\sqrt{3(x+h)+1} + \sqrt{3x+1})}$$

$$\frac{3}{\sqrt{3(x+0)+1} + \sqrt{3x+1}} \rightarrow \frac{3}{2\sqrt{3x+1}}$$

33. Given the function $f(x) = x^4 - 4x^3$, find:

- the zeros of the function
- the critical points and the intervals of increasing and decreasing.
- Any possible inflection points and intervals of concave up or concave down.
- Finally, sketch the graph. Use your analysis from the 1st and 2nd derivative tests and the zeros you found.

34.



Note: This is the graph of the derivative of f , not the graph of f .

The figure above shows the graph of f' , the derivative of a function f . The domain of f is the set of all real numbers x such that $-10 \leq x \leq 10$.

- For what values of x does the graph of f have a horizontal tangent?
- For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.
- For what values of x is the graph of f concave downward?

$$10 \quad f(x) = 5x + 2\sqrt[3]{x} - \frac{3}{x^2}$$

$$f(x) = 5x + 2x^{\frac{1}{3}} - 3x^{-2}$$

$$f'(x) = 5 + \frac{2}{3}x^{-\frac{2}{3}} + 6x^{-3}$$

$$f'(x) = 5 + \frac{2}{3\sqrt[3]{x^2}} + \frac{6}{x^3}$$

$$11) f(x) = \sin^2(3x+1)$$
$$f(x) = (\sin(3x+1))^2$$

$$f'(x) = 2(\sin(3x+1))(\cos(3x+1)) \cdot 3$$

$$f'(x) = 6(\sin(3x+1))(\cos(3x+1))$$

$$12) f(x) = \ln(\sin(x)) \rightarrow \frac{\cos(x)}{\sin(x)} \rightarrow \cot(x)$$

$$13. f(x) = \ln(\sqrt{2x+3})$$

$$f(x) = \ln(2x+3)^{\frac{1}{2}}$$

$$f'(x) = \frac{\frac{1}{2}(2x+3)^{-\frac{1}{2}} \cdot 2}{(2x+3)^{\frac{1}{2}}}$$

$$= \frac{1}{(2x+3)^{\frac{1}{2}} \cdot (2x+3)^{\frac{1}{2}}}$$

$$= \frac{1}{\sqrt{2x+3} \cdot \sqrt{2x+3}} = \frac{1}{2x+3}$$

$$14. f(x) = \frac{e^{2x}}{x^2}$$

$$\begin{aligned} f'(x) &= \frac{x^2 \cdot e^{2x} \cdot 2 - e^{2x} (2x)}{(x^2)^2} \\ &= \frac{2x^2 e^{2x} - (2x)e^{2x}}{x^4} \\ &= \frac{2x e^{2x} - 2e^{2x}}{x^3} \end{aligned}$$

$$15. f(x) = \sqrt[4]{(x^2 + 5x)^3}$$

$$f(x) = (x^2 + 5x)^{3/4}$$

$$f'(x) = \frac{3}{4}(x^2 + 5x)^{-1/4}(2x + 5)$$

$$= \frac{6x + 15}{4 \sqrt[4]{x^2 + 5x}}$$

$$16. f(x) = \sqrt{x} \tan x$$

$$f(x) = x^{\frac{1}{2}} \tan x$$

$$\begin{aligned} f'(x) &= \sqrt{x} \sec^2 x + \frac{1}{2} x^{-\frac{1}{2}} \tan x \\ &= \sqrt{x} \sec^2 x + \frac{\tan x}{2\sqrt{x}} \end{aligned}$$

$$17. f(x) = x^3 \sec(e^{3x} - 1)$$

$$f'(x) = x^3 \cdot \sec(e^{3x} - 1) \tan(e^{3x} - 1) (e^{3x} \cdot 3) + 3x^2 \sec(e^{3x} - 1)$$

$$18. f(x) = e^{\sqrt{2x}} \quad \sqrt{2x} \rightarrow (2x)^{\frac{1}{2}}$$

$$\begin{aligned} f'(x) &= e^{\sqrt{2x}} \cdot \frac{1}{2} (2x)^{-\frac{1}{2}} \cdot 2 \\ &= \frac{e^{\sqrt{2x}}}{(2x)^{\frac{1}{2}}} = \frac{e^{\sqrt{2x}}}{\sqrt{2x}} \end{aligned}$$

<u>function</u>	<u>Derivative</u>
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\sec^2 x$
$\sec x$	$\sec x \tan x$
$\csc x$	$-\csc x \cot x$
$\cot x$	$-\csc^2 x$

19.

$$y = x^2 \cdot \ln(3x)$$

$$y' = x^2 \cdot \frac{3}{3x} + 2x \ln(3x)$$

$$= (1)^2 \cdot \frac{3}{3(1)} + 2(1) \ln(3)$$

$$= 1 + 2 \ln(3)$$

$$20. \quad y = 3x^2 - 2x + 1$$

$$\begin{aligned} x &= -1 \\ y &= 3(-1)^2 - 2(-1) + 1 \\ &= 3 + 2 + 1 = 6 \end{aligned}$$

$$(-1, 6)$$

$$\begin{aligned} y' &= 6x - 2 \\ &= 6(-1) - 2 \\ &= -6 - 2 = -8 \end{aligned}$$

$$y - 6 = -8(x + 1)$$

21)

$$y = 5 - x^2$$

$$\begin{aligned} y' &= -2x \\ &= -2(2) \\ &= -4 \end{aligned}$$

$$x = 2$$

$$y = 5 - (2)^2$$

$$y = 1$$

$$(2, 1)$$

$$y - 1 = -4(x - 2)$$

$$22) s(t) = 2t^3 - 15t^2 + 24t - 10$$

$$v(t) = 6t^2 - 30t + 24$$

$$a(t) = 12t - 30$$

$$a) s(3) = -19$$

$$b) v(3) = -12$$

$$c) a(3) = 6$$

d) object is slowing down
 b/c $v(3)$ & $a(3)$ have
 diff. signs.

$$e) v(t) = 0$$

$$\begin{aligned} 0 &= 6t^2 - 30t + 24 \\ &= 6(t^2 - 5t + 4) \\ &= \frac{6(t-4)(t-1)}{t=4 \quad | \quad t=1} \end{aligned}$$

$$f) v(t) \begin{array}{c} + \quad - \quad + \\ \hline | \quad | \\ 1 \quad 4 \end{array}$$

$$[0, 1) \cup (4, \infty)$$

$$g) \quad |s(0) - s(1)| + |s(3) - s(1)|$$

31 units

$$23) h'(x) = f'(g(x)) \cdot g'(x)$$

$$h'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(1) \cdot (-1)$$

$$= 2 \cdot (-1) = -2$$

$$s'(x) = f(x)g'(x) + f'(x)g(x)$$

$$= f(1)g'(1) + f'(1)g(1)$$

$$= (2)(-1) + (2)(1)$$

$$= 0$$