24. The following table records the values of $f, f', g, and\ g'$ at $x=1$, $x=2$, and $x=3$.

x	$f(x)$	$f'(x)$	$g(x)$	$g'(x)$
1	2	1	2	3
2	5	4	3	4
3	0	6	-1	-2

If $\boxed{n(x) = \frac{f(x)}{g(x)}}$, $h(x) = f(g(x))$, find the value of each of the following: a) $n'(2)$ b) $h'(1)$

a) $n'(2) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

$\frac{g(2)f'(2) - f(2)g'(2)}{(g(2))^2}$

$\frac{3(4) - 5(4)}{(3)^2}$

$\frac{12-20}{9} \rightarrow \boxed{\frac{-8}{9}}$

b) $h'(1) = f(g(x))$

$= f'(g(x)) \cdot g'(x)$

$= f'(g(1)) \cdot g'(1)$

$= f'(2) \cdot (3)$

$= 4 \cdot 3$

$= \boxed{12}$

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25)

$$f(x) = \sqrt[3]{(x^2-2x-1)^2}$$

$$f(x) = (x^2-2x-1)^{2/3}$$

$$f'(x) = \frac{2}{3}(x^2-2x-1)^{-1/3}(2x-2)$$

$$f'(x) = \frac{2(2x-2)}{3\sqrt[3]{x^2-2x-1}}$$

$$f'(0) = \frac{2(2(0)-2)}{3\sqrt[3]{0^2-2(0)-1}} \rightarrow \frac{-4}{3\sqrt[3]{-1}}$$

$$\rightarrow \frac{-4}{-3} \rightarrow \frac{4}{3}$$

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28) $x^3+y^3=18y$

$$3x^2+3y^2\frac{dy}{dx}=18\frac{dy}{dx}$$

$$3x^2=18\frac{dy}{dx}-3y^2\frac{dy}{dx}$$

$$\frac{3x^2}{18-3y^2}=\frac{\frac{dy}{dx}\cancel{(18-3y^2)}}{\cancel{18-3y^2}}$$

$$\frac{x^2}{6-y^2}=\frac{dy}{dx}$$

29) $x^2y - xy^2 = 4x$

$$x^2\frac{dy}{dx} + y\,2x - \left(x2y\frac{dy}{dx} + y^2(1)\right) = 4$$

$$x^2\frac{dy}{dx} + 2xy - 2xy\frac{dy}{dx} - y^2 = 4$$

$$x^2\frac{dy}{dx} - 2xy\frac{dy}{dx} = 4 - 2xy + y^2$$

$$\frac{\frac{dy}{dx}\cancel{(x^2 - 2xy)}}{\cancel{x^2 - 2xy}} = \frac{4 - 2xy + y^2}{x^2 - 2xy}$$

3d) $x^2 - xy = y^2 + 1$

$y = 1$

$x^2 - x(1) = (1)^2 + 1$

$Q1$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad \cancel{x = -1}$

$(2,1)$

$2x - \left(x\frac{dy}{dx} + y(1)\right) = 2y\frac{dy}{dx}$

$2x - x\frac{dy}{dx} - y = 2y\frac{dy}{dx}$

$2x - y = 2y\frac{dy}{dx} + x\frac{dy}{dx}$

$\frac{2x-y}{2y+x} = \frac{\frac{dy}{dx}\cancel{(2y+x)}}{\cancel{2y+x}}$

$\frac{2(2)-1}{2(1)+2} \rightarrow \frac{3}{4}$

$y - 1 = 3/4(x-2)$

(33) $f(x)=x^4-4x^3$

a) $0=x^4-4x^3$

$=x^3(x-4)$

b) $=x=0,4$

$f'(x)=4x^3-12x^2$

$0=4x^2-12x^2$

$0=4x^2(x-3)$

$x=0,3$

$$f'(x)\quad \underset{-}{} \;|_{0}\; \underset{-}{} \;|_{3}\; \underset{+}{}$$

rel. min

Decreasing $(-\infty,0)\cup(0,3)$

Increasing $(3,\infty)$

c) $f''(x)=12x^2-24x$

$=12x(x-2)$

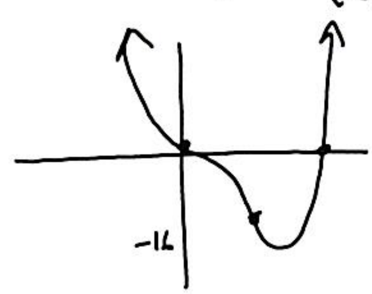
$x=0,2$

$$f''\quad + \;|_{0}\; - \;|_{2}\; +$$

Concave up $(-\infty,0)\cup(2,\infty)$

Concave down $(0,2)$

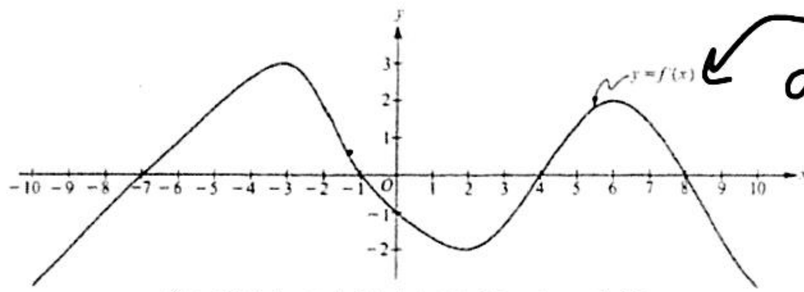
Inflection pts: $(0,0),(2,-16)$



33. Given the function $f(x) = x^4 - 4x^3$, find:

a) the zeros of the function
b) the critical points and the intervals of increasing and decreasing.
c) Any possible inflection points and intervals of concave up or concave down.
d) Finally, sketch the graph. Use your analysis from the 1st and 2nd derivative tests and the zeros you found.

34.



graph of the derivative

Note: This is the graph of the derivative of f, not the graph of f.

The figure above shows the graph of f', the derivative of a function f. The domain of f is the set of all real numbers x such that $-10 \le x \le 10$.

(a) For what values of x does the graph of f have a horizontal tangent?

$x = -7, -1, 4, 8$

(b) For what values of x in the interval $(-10, 10)$ does f have a relative maximum? Justify your answer.

where $f'(x)$ goes from a + to a −

$x = -1, 8$

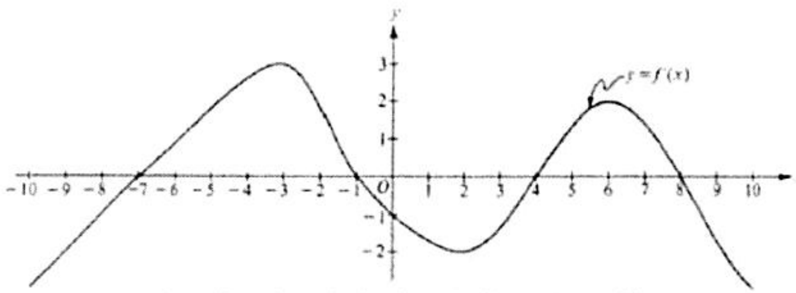
above x-axis to below x-axis

(c) For what values of x is the graph of f concave downward?

look where $f'(x)$ has a neg. slope

$(-3, 2) \cup (6, 10)$

34.



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